GROUP	FAMILY NAME	FIRST NAME

FIMI SCAN 2nd

Mechanics of Systems – End of semester 1 exam

Date 05/02/2025 – Duration 2h (14h-16h)

Kinematic analysis of a quick-return mechanism

Authorised documents: 2-page A4 personal formula sheet; calculator; table of standard joints.
Parts 1, 2 and 3 are independent. The marking scheme is indicative.

1 ANALYTICAL STUDY (~10 MARKS)

The system under consideration is shown in Figure 1 and its kinematical model is represented in **Figure 4**. The problem is planar of normal $\vec{z} = \vec{z}_{0,1,2,3,4}$

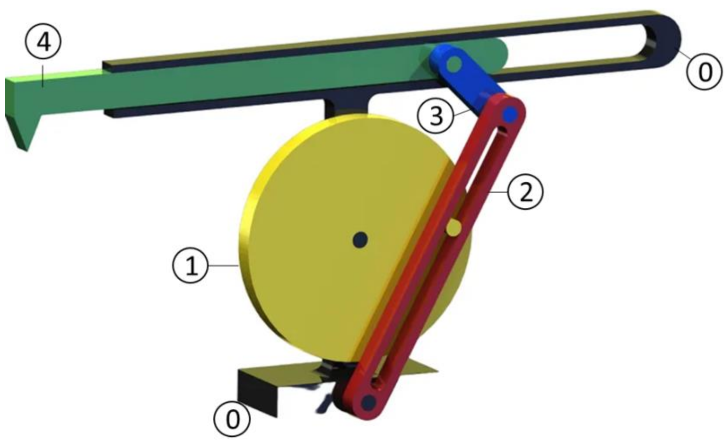

Figure 1 – CAD model of a quick-return mechanism

The mechanism comprises 4 rigid solids:

- A crankshaft S_1 of radius r connected to the ground S_0 via a revolute joint of axis $(B, \vec{z})$.

  Motion parameter for 1/0 : $\theta_1 = (\vec{x}_0, \vec{x}_1)$.
- An oscillating arm S_2 of length L:
  - connected to the ground S_0 via a revolute joint of axis $(O, \vec{z})$

    Motion parameter for 2/0 : $\theta_2 = (\vec{x}_0, \vec{x}_2)$.
  - **and** connected to rod S_3 via a revolute joint of axis $(C, \vec{z})$

    No parameter for this joint
  - **and** connected to crankshaft S_1 so that a pin at the tip of S_1 slides within the groove of S_2 of axis $(O, \vec{x}_2)$

    No parameter for this joint.

- A rod S_3 of length *e* connected to the oscillating arm S_2 via a revolute joint of axis $\left(C,\vec{z}\right)$ (see above) and connected to slider S_4 via a revolute joint of axis $\left(D,\vec{z}\right)$

  No parameter for the motion of rod 3.

- A slider S_4, a) connected to S_3 via a revolute joint of axis $\left(D,\vec{z}\right)$ with no parameter for this joint and, b) connected to the ground S_0 via a prismatic joint of axis $\left(D,\vec{x}_{4,0}\right)$.

  Motion parameter for 4/0 : $x = \vec{OD}.\vec{x}_{0,4}$

The geometrical data are given in Figure 4. The input motion is that of the crankshaft and the output motion is the slider motion with respect to the ground.

1.1 CONSTRAINT EQUATIONS

1. Represent the graph of links and the change of basis diagrams in the box under **Figure 4**.
2. Write and develop the constraint equation(s) associated with the joint between S_1 and S_2. *Do not try to solve.*

3. Write and develop the constraint (or 'joint') equation(s) associated with the solid between S_2 and S_4. *Do not try to solve.*

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4. Give the degree of mobility of the system.

1.2 KINEMATIC SCREWS (WRENCHES)

Having established the relationship between the kinematic parameters, this section deals with the study of the various motions.

5. Give the nature of motion 1/0 and specify the nature of the associated screw. Express the screw coordinates (sum and moment) at point A.

6. Give the sum and moment of the screw associated with motion 2/0 at point O. Define the trajectory of point C in its motion with respect to S_0. Calculate the velocity vector $\vec{V}(C,2/0)$ and the acceleration vector $\vec{A}(C,2/0)$

7. Express the screw coordinates (sum and moment) for motion 1/2 at point A. Verify that $\vec{V}(A,1/2)$ is in the $\vec{x}_2$ direction (*it is suggested to use the time-derivative of one constraint equation*)

8. Slider S_4 is the output of the mechanism, give the nature of motion 4/0 and express the associated screw coordiantes (sum and moment) at point E.

1.3 QUICK RETURN MECHANISM

A simplified analysis is proposed in order to appraise the speed difference between the forward stroke (in the direction of $-\vec{x}_{0,4}$) and the backward stroke (in the direction of $+\vec{x}_{0,4}$).

9. Figure 2 below shows the translation parameter x versus the crankshaft angle θ_1 obtained by numerical solution of one constraint equation using the following numerical data L=500 mm ; e=120 mm ; r=100 mm ; $b+d$=500 mm.

   a- Assuming that the crankshaft rotational speed $\omega_1 = \dot{\theta}_1$ is constant, explain how the mean speeds of the slider for the backward and forward strokes can be derived from the indications in Figure 2.

   b- Determine these mean speeds for $\omega_1 = 0.6$ rad/s and conclude on the quick return property of the system.

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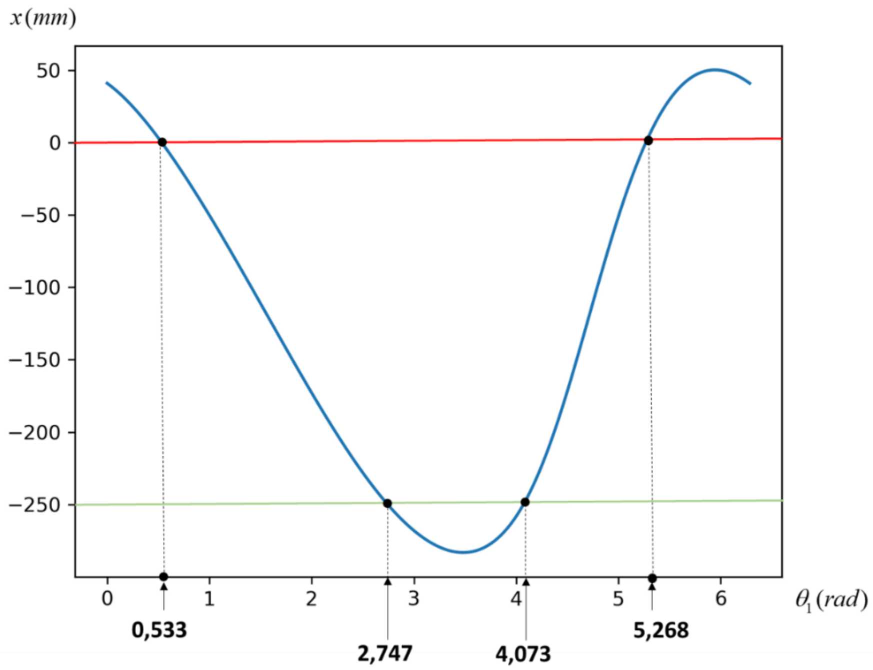

Figure 2 – Slider parameter x versus crankshaft angle θ_1

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2 QUIK RETURN MECHANISM : GRAPHICAL ANALYSIS (~6 MARKS)

For all the questions in this section, the graphical constructions **will be represented in Figure 5 (annex 1)** and the associated justifications will be given in the **answer boxes** below.

a) $\vec{V}(A,1/0)$ is given in Figure 5 (Annex 1), justify the direction of this vector.

b) Using the combination of velocities, draw $\vec{V}(A,2/0)$ and $\vec{V}(A,1/2)$

c) Deduce $\vec{V}(C/0)$

d) Find the position of the instant centre I_{30} for motion 3/0. Draw the velocities $\vec{V}(D/0)$ and $\vec{V}(E/0)$

e) Draw $\vec{V}(O,2/1)$ and deduce the position of the instant centre I_{21} for motion 2/1

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3 ADDITION OF A ROLLER – ROLLING WITH NO SLIPPING (~4 MARKS)

In order to minimise sliding between solids 1 and 2, a roller of radius ρ (disc, solid 5) is mounted on the pin at A (Figure 3). A revolute joint of axis $(A, \vec{z})$ connects solids 5 and 1, the associated motion parameter is $\alpha = (\vec{x}_1, \vec{x}_5)$. For the particular position under consideration, it is assumed that the contact between the disc (solid 5) and the groove of solid 2 occurs at point I only (one-sided contact on the left wall of the groove while non-represented clearance is supposed to exists on the opposite side). This contact at point I is supposed to be permanent and <u>with no slipping.</u>

1 – Having determined a unit normal vector to the contact at point I, derive the pitching (spin) and rolling vectors $\vec{P}(I,5/2)$ et $\vec{R}(I,5/2)$.

2 – Write and develop the no-slipping condition at the contact point I

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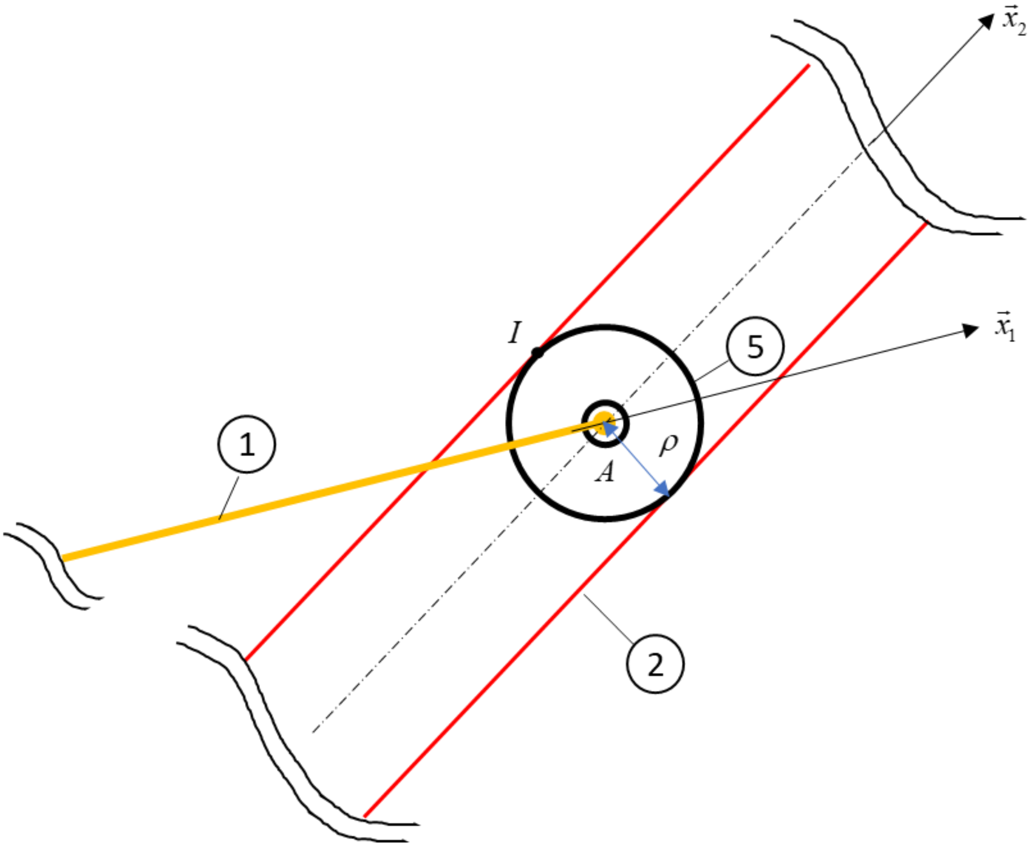

Figure 3 – Disc-groove contact

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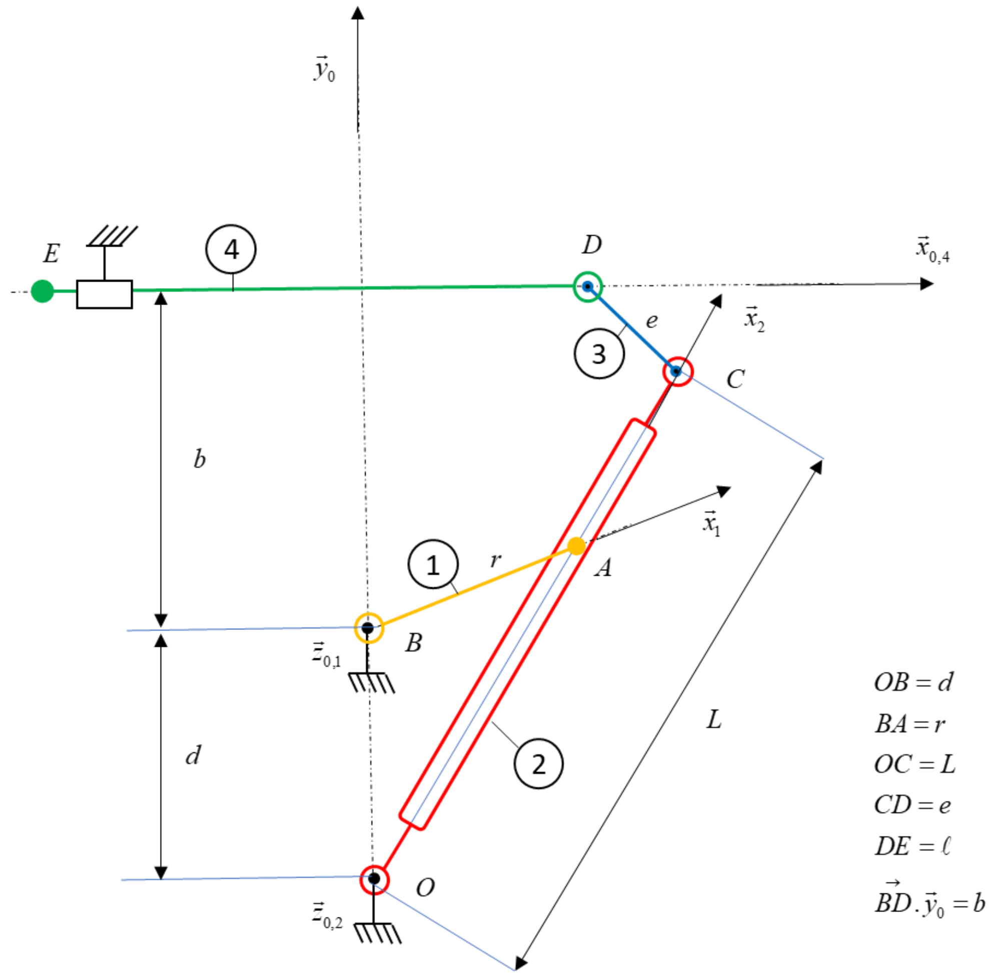

Figure 4. Kinematic model of the quick-return mechanism

Change of basis diagrams and graph of links

ANNEX 1 - GROUP :	FAMILY NAME	FIRST NAME

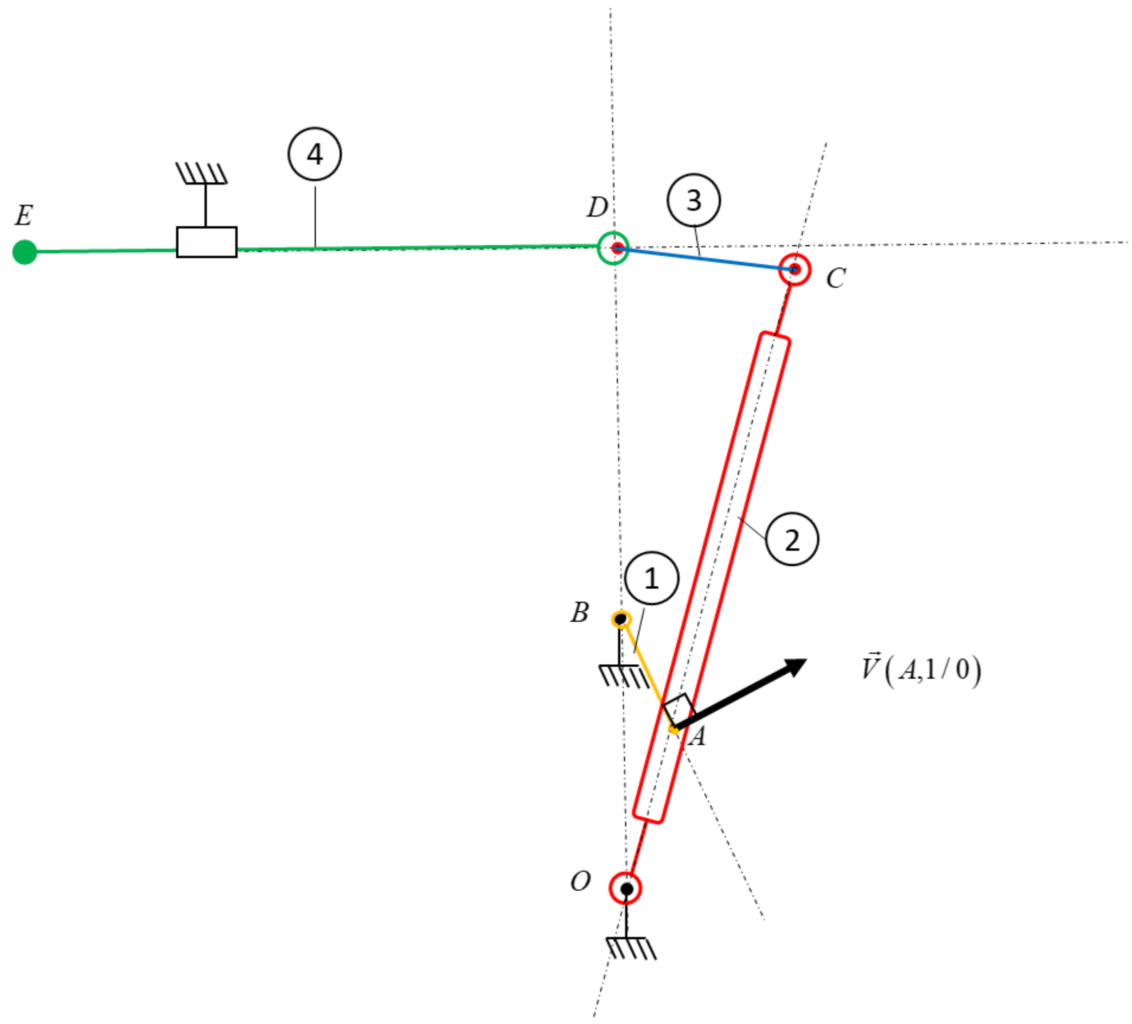

Figure 5 : Figure for graphical construction