End of First Semester Examination

Monday 24th January 2022 - Time Allowed: 3 hours

Instructions:

*Make sure your work is **well presented** and **readable**. Calculators and a formula sheet (one double-sided A4 page) allowed. The sections of the test are independent and can be attempted in any order. **Un-justified answers may not be taken into account.***

Part 1: Wireless charging (≈ 9.5 pts)

*Wireless charging of electronic devices has recently developed for connected watches or cellphones. This technology is based on static induction. The inductor is a **planar coil** (exciting coil or primary) located in the charging base (bottom left part in Fig1) whereas the induction receiver is a **planar coil** (receptor coil or secondary) located inside the watch. The right part of Fig.1 models the system using an exciting planar coil (the charging base) of radius R and a receptor planar coil (the watch) of radius a and same axis (Oz). The two coils are separated by the distance d.*

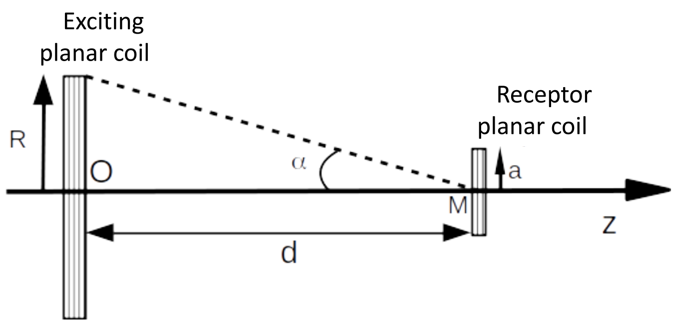


Figure 1: Left: Example of wireless-charging base for a connected watch – Right: Sketch of the charging system

1. Study of the mutual induction between the exciting coil and the receptor coil

Note: useful numerical data as well as formulas are given at the end of the exercise.

*The magnetic field created by a **planar** coil (or loop, of negligible thickness) having N_1 turns, circular, of axis (Oz) (and normal $+\overrightarrow{u_z}$), of centre O and radius R, crossed by a current $i_1(t)$ can be given at a point M located on its axis by:*

$$\overrightarrow{B_1} = \frac{\mu_0 N_1 i_1}{2R} \cdot sin^3(\alpha) \cdot \vec{u}_z$$

where the angle α is given in Fig.1.
*We consider here a **cylindrical** coordinate system $(O, \overrightarrow{u_r}, \overrightarrow{u_\theta}, \overrightarrow{u_z})$.*

Question 1: Considering a point M located **on the coil's axis (Oz)**, justify the direction of the magnetic field $\overrightarrow{B_1}$.

*A second **planar coil** (or loop, of negligible thickness), which is the receptor (or induction receiver) coil, is placed at point M. It has N_2 turns, its radius is a << R and its axis is (Oz). Its surface $S_2 = \pi a^2$ is so small that the magnetic field $\overrightarrow{B_1}$ can be considered uniform over its surface S_2 and equal to the value of the magnetic field on the (Oz) axis. Note that the surface S_2 is oriented by the vector $+\overrightarrow{u_z}$.*

Question 2: Prove that the coefficient of mutual induction between the two coils can be written:

$$M = \frac{\mu_0 \pi a^2 N_1 N_2 R^2}{2\left(\sqrt{R^2 + d^2}\right)^3}$$

The receptor coil is connected to a battery which will be modelled by a resistance R_S. The total resistance (excluding R_S) of the receptor circuit is denoted R_2 and the self-inductance of the receptor coil is named L_2.

Question 3: Prove that the **self-inductance** of the receptor coil can be written:

$$L_2 = \frac{\mu_0 \pi a N_2^2}{2}$$

To answer this question, we will consider that when the receptor coil is crossed by a current $i_2(t)$, the resulting magnetic field $\overrightarrow{B_2}$ is *of the same type* as the one created by the exciting coil. The magnetic field $\overrightarrow{B_2}$ will be considered uniform over the whole surface of the receptor coil.

Question 4: Numerical application: calculate L_2.

Question 5: Write the expression of the mutual induction's e.m.f. e_{12} that comes up in the receptor coil as a function of M and i_1.

Question 6: Make a sketch of the electric circuit which is equivalent to the receptor system. On this sketch show explicitly the mutual induction's e.m.f. e_{12} in the form of a voltage generator. Write the Ohm-Kirchoff's equation that applies in this circuit.

2. Study and optimization of the system when supplied with alternative current

The current in the exciting coil is given by: $i_1(t) = I_1 cos(\omega t)$. The exciting coil is supplied by a voltage of frequency 10 kHz. From now on it is possible to use the complex notation for the electric quantities.

Question 7: By using the value for L_2 calculated in Question 4, justify that the impedance related to L_2 can be neglected with respect to the resistances of the receptor circuit.

From now on, the self-inductance L_2 of the receptor coil will be neglected.

Question 8: Give the complex expression of the voltage $\underline{e_{12}}$ in harmonic (sinusoidal) regime as a function of M, ω and i_1.

The magnetic field $\overrightarrow{B_1}$ due to the inductor coil creates in the whole space an electric field $\overrightarrow{E_2}$ which can be expressed in the vicinity of the induction receiver (or receptor) coil (and consequently close to the (Oz) axis): $\overrightarrow{E_2} = E_{2\theta}(r).\overrightarrow{u_\theta}$. We assume that this electric field is nil on the (Oz) axis.
To simplify the notations, we introduce η (eta) such that: $\eta = \frac{R^2}{\left(\sqrt{R^2+d^2}\right)^3}$

Question 9: Find again the value of $\underline{e_{12}}$ by applying one of Maxwell's equations. Indication: find the electric field $\overrightarrow{E_2}$ induced by $\overrightarrow{B_1}$. Then deduce the potential difference generated by $\overrightarrow{E_2}$ in the induction receiver (or receptor) coil of circumference $2\pi a$ (located at $r=a$ from the *(Oz)* axis) and made of N_2 turns.

Question 10: The current $i_2(t)$ coming up in the receptor coil is expressed as: $i_2(t) = I_2 cos(\omega t + \varphi_2)$ where I_2 is the amplitude of $\underline{i_2}$. Determine I_2 and φ_2 by using the equation established in Question 6 and the complex notation.

Question 11: Express the instantaneous power P and subsequently the mean power <P> available at the terminals of R_S. To do that, recall that $\langle sin^2(t)\rangle = \frac{1}{2}$. Give the numerical value of the mean power.

Question 12: Is it possible to neglect the wave nature and the propagation of the electromagnetic field?

*To reduce the charging times, we embed **the receiver coil** in a ferromagnetic material of permeability $\mu_r = 5000$.*

Question 13: Explain why this action increases the power available at the terminals of R_S. Give a rough estimate of the gain obtained by this action (complicated calculations are not necessary).

Question 14: Which other parameter (apart from embedding the receiver coil in a ferromagnetic material) can we control in a realistic way to increase the power exchanged between the exciting and receiver coils? In your answer you should specify how these parameter changes will impact on the design of the watch and its charging station.

Question 15: (Bonus) If the battery's capacity is 300 mAh, it is charged with a voltage of 1,5V and assuming that all electrical power available over R_S is stored in the battery, how much time will it take to charge it based on the afore mentioned numerical data? Provide the calculation for both a receiver coil embedded in air or in the ferromagnetic material (neglect L_2in this case). Additional information: a 300 mAh battery can deliver 300 mA at 1,5V during 1 hour.

Numerical data: $\mu_0 = 4\pi \cdot 10^{-7} H/m$, $N_1 = 10$, $N_2 = 5$, $R = 10$ cm, $d = 2$ mm, $a = 5$ mm, $R_2 = 10^{-1}\Omega$, $R_S = 10\ \Omega$, $I_1 = 1$ A, and $\omega = 2\pi \cdot 10^4$ rad/s.

Formulas: curl (rotational) in cylindrical coordinates:

$$\overrightarrow{rot}(\vec{E}) = \nabla \times \vec{E} = \left(\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z}\right)\overrightarrow{u_r} + \left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right)\overrightarrow{u_\theta} + \left(\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) - \frac{1}{r}\frac{\partial E_r}{\partial \theta}\right)\overrightarrow{u_z}$$

Part 2: Motion contactless capacitive sensor (≈ 5.5 pts)

Motion sensors require a contactless measurement to not disturb the movement of the observed piece. One solution to this problem is to rely on a plane capacitor structure, in a configuration analogous to the one seen in Figure 2. The piece whose movement we wish to measure slides between two plates of a capacitor.
On Fig. 2, we consider a capacitor composed of two rectangular conductor plates, which we will consider perfectly planar and of negligible thickness, with width W and length L. The plates are connected to a potential difference V. Between the plates, a piece of material with absolute permittivity ϵ, with the same width W but length $L/2$, occupies the right side of the space in between the plates (the one defined by $0 < x < L/2$). The other half of the capacitor is empty.
During the analysis of the symmetries and invariances we shall consider that both capacitor plates are infinite (even if, for the calculations of the capacitances, we will consider them not to be infinite).

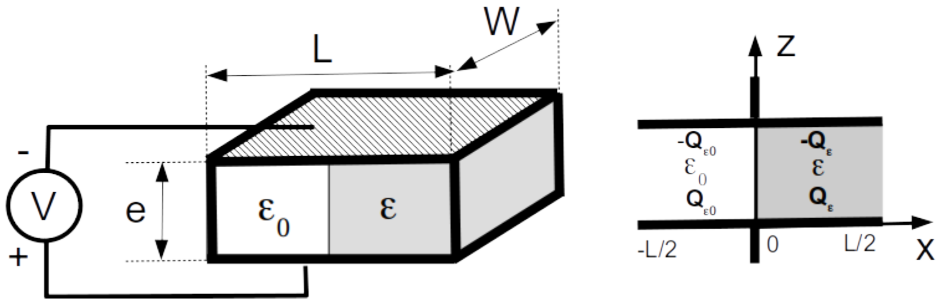


Figure 2: Capacitor constituted of two parts, one empty and the other occupied by a dielectric with permittivity ϵ

When the voltage source V is turned on, the uniform surface charge that will appear on the plates (σ_{ϵ_0} on one side and σ_ϵ on the other) creates an electric field $\overrightarrow{E_{\epsilon_0}}$ in the empty side and $\overrightarrow{E_\epsilon}$ in the side filled with the dielectric piece. Let us consider that both electric fields are in the $\overrightarrow{u_z}$ direction, and can be given by: $\overrightarrow{E_{\epsilon_0}} = A_{\epsilon_0}\overrightarrow{u_z}$ and $\overrightarrow{E_\epsilon} = A_\epsilon\overrightarrow{u_z}$.

Question 16: Prove **rigorously** that A_{ϵ_0} and A_ϵ are constants.

The metal plates of the capacitor carry a total charge Q and $-Q$, respectively. This charge will be divided between Q_{ϵ_0} , the charge stored in the half of the capacitor plate defined by $x < 0$ (with a surface charge density σ_{ϵ_0}), and

Q_ϵ, the charge stored in the other half of the capacitor plate, defined by $x > 0$ (with a surface charge density σ_ϵ). Therefore, $Q = Q_{\epsilon_0} + Q_\epsilon$.

Question 17: Give the expression of the fields $\overrightarrow{E_{\epsilon_0}}$ and $\overrightarrow{E_\epsilon}$, first as a function of the densities of charge σ_{ϵ_0} and σ_ϵ and the permittivities ϵ_0 and ϵ, and then as a function of Q_{ϵ_0}, Q_ϵ, W, L and ϵ_0 and ϵ.

This system can be modelled as the association of two capacitors $C_{1\epsilon_0}$ and $C_{1\epsilon}$ connected in parallel to the same voltage V. $C_{1\epsilon_0}$ is the capacitance of the part of the capacitor whose space in between the plates is empty ($x < 0$) while $C_{1\epsilon}$ is the one filled with the dielectric piece ($x > 0$).

Question 18: Calculate $C_{1\epsilon_0}$ and $C_{1\epsilon}$ as a function of W, L, ϵ_0 and ϵ. Then give, with proper justification, the total capacitance C_1 of the system, first as a function of $C_{1\epsilon_0}$ and $C_{1\epsilon}$, then as a function of W, L, ϵ_0 and ϵ.

In reality, the piece whose movement we wish to measure is of length L and not $L/2$ and it is contained between two capacitors of capacitance C_1 and C_2. At rest, half of the piece is located in the first capacitor and the other half in the second capacitor. The piece is perfectly centred when at rest, meaning that it occupies the space between $x = 0$ and $x = L$ (see Fig. 3). The space between both capacitors can be neglected. Hence, when at rest, $C_1 = C_2$.
When the piece moves of a total distance x (it can only move in the (Ox) direction), the capacitance of each capacitor changes, and become $C_1' = C_{1\epsilon_0}' + C_{1\epsilon}'$ and $C_2' = C_{2\epsilon_0}' + C_{2\epsilon}'$.

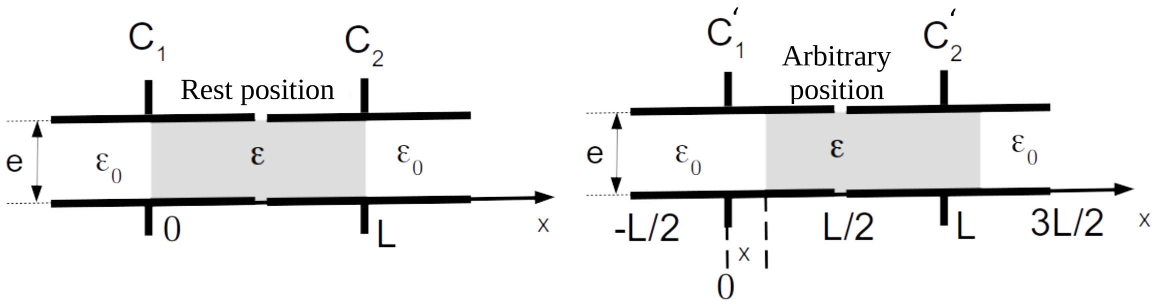


Figure 3: Left: at rest: the piece is perfectly centered between both capacitors. Right: the piece has moved to the right by an amount x on the (Ox) axis.

Question 19: Based on question 18, write the total capacitance **at rest** (dielectric piece perfectly centred between both capacitors) of the second capacitor: $C_2 = C_{2\epsilon_0} + C_{2\epsilon}$, as a function of W, L, ϵ_0, e and ϵ.

Question 20: Give the new capacitances C_1' and C_2' of both capacitors when the piece moves by a distance x, as well as the variations of both of the capacitances $\Delta C_1 = C_1' - C_1$ and $\Delta C_2 = C_2' - C_2$, between the rest position and another position given by x. Comment on the result.

These two capacitors are a part of a circuit, powered by an alternating current and represented in Fig. 4. The measurement voltage $\underline{V_m}$ in the circuit is given by $\underline{V_m} = \underline{U}\left(\frac{1}{2} - \frac{\underline{Z_2}}{\underline{Z_1}+\underline{Z_2}}\right)$, where $\underline{Z_1}$ and $\underline{Z_2}$ are the complex impedances of both capacitors, respectively.

Question 21: Bonus: demonstrate the expression for $\underline{V_m}$ given above.

Question 22: In the case where the voltage U is **sinusoidal** with an angular frequency ω, express $\underline{V_m}$ as a function of the capacitances C_1', C_2' and U, then as a function of ΔC_1, C_1 and $\underline{U}$.

Question 23: Deduce from this the expression for the voltage $\underline{V_m}$ for any position of the movable piece, as a function of x, L, $\underline{U}$ et the permittivities.

Question 24: Numerical application: Calculate the resting capacitances C_1 and C_2, as well as ΔC_1 once the movable piece has translated by $x = 1$ *cm*. If the voltmeter that measures $\underline{V_m}$ has a limiting sensitivity of 1mV (i.e. it cannot measure variations in voltage below this limit), what is the smallest translation of the movable piece that can be detected? How could this system be improved?

Numerical data: $\epsilon_0 = \frac{1}{36\pi}10^{-9}F/m$, $\epsilon_r = 5$, *W = 20 cm, L = 2 cm, e = 5cm, amplitude of the voltage* $\underline{U}$ *= 10V.*

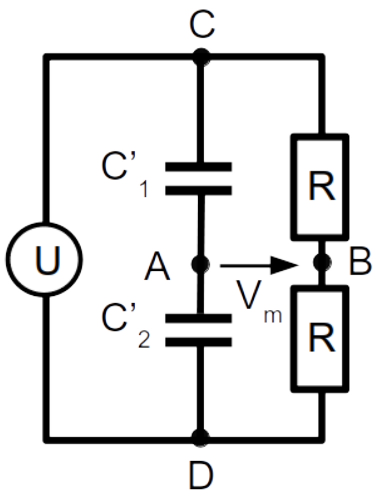


Figure 4: Measurement circuit

Part 3: Propagation of a thermal wave (≈ 5 pts)

We give: $div\left(\overrightarrow{grad}(V)\right) = \Delta V$, and we recall the expression of the Laplacian operator in Cartesian coordinates:

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

In a material that conducts heat, the thermal (heat) flux $\vec{j}$ *follows Fourier's law:*

$$\vec{j} = -\lambda \overrightarrow{grad}(\Theta) \quad (1)$$

where λ *is the thermal conductivity (W.m*$^{-1}$*.K*$^{-1}$*) and* Θ *the temperature. In the absence of a heat source, conservation of heat leads to the following equation of conservation:*

$$\frac{\partial}{\partial t}\left(\rho . C_p . \Theta\right) + div(\vec{j}) = 0 \quad (2)$$

where ρ *is the density of the material and* C_p *its specific heat capacity (J.kg*$^{-1}$*.K*$^{-1}$*).*

Question 25: Show that the temperature in this type of material follows a law of the form:

$$D.\Delta\Theta - \frac{\partial \Theta}{\partial t} = 0 \quad (3)$$

Give the expression of *D*. We call this parameter the **thermal diffusivity**. What is its dimension?

We are interested in the propagation of a thermal wave, of a type referred to as « forced-sinusoidal in temperature », in soil which is considered to be a semi-infinite material for which z ≤ 0 (z axis oriented upwards). We make the hypothesis that the temperature wave generated in the soil can be written in complex form:

$$\underline{\Theta}(z,t) = \Theta_0 e^{j(\omega t + kz)} \quad (4)$$

The thermal wave represented by equation 4 is the solution to equation 3.

Question 26: Determine the possible value(s) of *k* and express them in the form of a complex number $k = \pm A(1-j)$. Give the expression of *A*.

Note: *the exercise can be continued from here by taking for k the following expression:* $k = \pm A(1-j)$.

Question 27: Chose and justify (remember that $z \leq 0$) the correct expression for *k* and give the real expression of the temperature wave $\Theta(z, t)$. Describe this wave as precisely as possible and give the expression of its velocity as a function of ω and *A*.

Question 28: We define the parameter δ as being the skin depth, i.e. the (positive) distance over which the wave amplitude is attenuated by a factor of *e* ($e = exp(1) \approx 2.71828$). Give the literal expression of this depth.

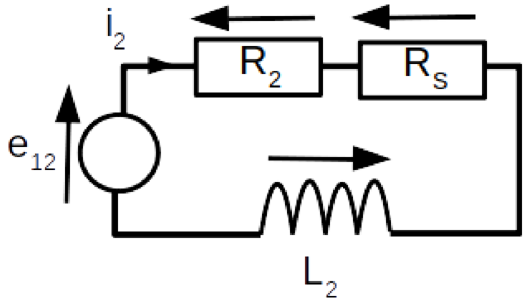
Question 29: We consider that the forced-sinusoidal temperature regime begins at $t = 0$ (no driving of temperature oscillations exists for $t < 0$). The temporal period of the wave is *T*, which can be kept in literal form in absence of information about ω.
— Schematically represent the wave at $z = 0$ as a function of time;
— Schematically represent the wave as a function of *z* at a time $t = 2T$ (it is recalled that the soil is located in the region $z<0$).
Indicate all useful information on your graphs.

Question 31: *Application:*
Given that the thermal diffusivity of soil and of materials like concrete and glass is comprised between 0.3×10^{-6} and 1×10^{-6} (SI units), estimate the order of magnitude of the depth at which a cellar (cave) must be situated so that it is not subjected to annual fluctuations in temperature. In this case, are daily temperature fluctuations observable?
This is an open question, you must therefore detail the different hypotheses that you make, estimate in a plausible manner any missing information, and explain your reasoning step by step.

Devoir de synthèse de Physique

2ème année, 24 Janvier 2022 : Correction et barême

Partie 1	Points	Total question
Q1 Tout plan contenant l'axe (Oz) est plan de d'antisymétrie de la distribution de courant.	1	
En tout point de cet axe, le champ magnétique $\vec{B}_1$,qui est contenu dans les plans d'antisymétrie, est donc le long de la direction (Oz).	1	2
Q2 Induction mutuelle. Calcul du flux envoyé par $\vec{B}_1$ dans la surface S_2 orientée le long de $\vec{u}_z$:		
$\phi_{12} = N_2 \iint_{S_2} \vec{B}_1.d\vec{S} = N_2 \iint B_1.dS = N_2 B_1 \pi a^2$	1	
(car $\vec{B}_1$est supposé uniforme sur toute la surface S_2) $\phi_{12} = \pi a^2 N_2 \frac{\mu_0 N_1}{2R} sin^3(\alpha)\, i_1 = M\, i_1$		
Donc $M = \pi a^2 N_2 \frac{\mu_0 N_1}{2R} sin^3(\alpha)$	2	
Avec $sin\, \alpha = \frac{R}{\sqrt{R^2+d^2}}$ on a :		
$M = \frac{1}{2}\mu_0 \pi a^2 N_1 N_2 \frac{R^2}{(R^2+d^2)^{3/2}}$	1	4
Q3 Induction propre L_2 : Calcul du champ magnétique $\vec{B}_2$ au centre de la bobine réceptrice et supposé uniforme sur toute la surface S_2 :		
on l'obtient à partir de la formule de $\vec{B}_1$ avec $d = 0$ et $\alpha = \frac{\pi}{2}$.	1	
$\vec{B}_2 = \frac{\mu_0 N_2 i_2}{2a}$	1	
On calcule alors le flux envoyé par $\vec{B}_2$ dans la bobine réceptrice :		
$\phi_{22} = N_2 \iint_{S_2} \vec{B_2}.d\vec{S}_2$	1	
$\phi_{22} = N_2 B_2 S_2 = \frac{\mu_0 \pi a^2 N_2^2}{2a} i_2$		
On entire : $L_2 = \frac{\mu_0 \pi a^2 N_2^2}{2a}$	2	5
Q4 A.N. $L_2 = 247\, nH$	2	2
Q5 Tension d'induction e_{12} : $e_{12} = -\frac{d\phi_{12}}{dt} = -M\frac{di_1}{dt}$	1	1
Q6		

	2	
$-M\dfrac{di_1}{dt} - (R_2 + R_S)i_2 - L_2\dfrac{di_2}{dt} = 0$	2	4
Q7 On calcule l'impédance de L_2 dans les conditions de l'énoncé :		
$Z_{L2} = jL_2\omega \Rightarrow \lvert Z_{L2}\rvert = 247.10^{-9}.2\pi.10^4 = 0.016\,\Omega$	1	
Si on compare à R_S et R_2 on est une décade en dessous de la plus faible, donc on peut négliger L_2.	1	2
Q8 En régime harmonique, avec la valeur de M calculée précédemment, on trouve :		
$\underline{e_{12}} = -j\omega M\underline{i_1}$	1	1
Q9 $\vec{\nabla} \wedge \vec{E}_2 = -\dfrac{\partial \vec{B}_1}{\partial t}$	1	
On simpifie le rotationnel avec les propriétés de symétrie données dans l'énoncé.		
$\dfrac{1}{r}\dfrac{\partial(r\underline{E_{2\theta}})}{\partial r} = -\dfrac{j\mu_0\omega NR^2}{2(R^2+d^2)^{3/2}}\,\underline{i_1} = -\dfrac{j\omega\mu_0 N_1}{2}\eta\underline{i_1}$	1	
$\dfrac{\partial(r\underline{E_{2\theta}})}{\partial r} = -\dfrac{j\omega\mu_0 N_1}{2}\eta\underline{i_1}r \Rightarrow r\underline{E_{2\theta}} = -\dfrac{j\omega\mu_0 N_1}{4}\eta\underline{i_1}r^2 + A(t)$		
$\underline{E_{2\theta}} = -\dfrac{j\omega\mu_0 N_1}{4}\eta\underline{i_1}r + \dfrac{A(t)}{r}$	2	
Avec les conditions initiales données ($\underline{E_{2\theta}}(r=0)=0$), on trouve $A(t) = 0$	1	
$\underline{E_{2\theta}} = -\dfrac{j\omega\mu_0 N_1}{4}\eta\underline{i_1 r}$		
Pour trouver e_{12} on fait circuler N_2 fois le champ E_{12} sur la spire :		
$\underline{e_{12}} = N_2\displaystyle\int_{spire}\underline{E_{2\theta}}\,rd\theta \;=\; -\dfrac{j\omega\mu_0 N_1 N_2}{4}\eta\underline{i_1}r^2\int_0^{2\pi} d\theta$	1	
avec $r = a$ puisque la spire est à r constant	1	
$\underline{e_{12}} = -\dfrac{j\omega\mu_0 N_1 N_2}{4}\eta\underline{i_1}a^2 2\pi = -j\dfrac{\mu_0\pi a^2 N_1 N_2}{2}\eta\omega\underline{i_1} = -jM\omega\underline{i_1}$	1	8
Q10 Détermination de $\underline{i_2}$		
D'après le circuit obtenu précédemment, et en négligeant L_2 comme indiqué, on a :		
$\underline{i_2} = -\dfrac{j\omega M\underline{i_1}}{R_S+R_2} = -j\omega\dfrac{\mu_0 N_1 N_2\pi a^2\eta}{2(R_S+R_2)}\,\underline{i_1} = I_2\,e^{j(\omega t+\varphi_2)}$	1	
$\lvert I_2\rvert = \dfrac{\omega M I_1}{R_S+R_2} = \dfrac{\mu_0\omega N_1 N_2\pi a^2\eta}{2(R_S+R_2)}\,I_1$	1	
$arg(\underline{i_2}) = \pi + arg(\underline{i_1}) + \pi/2 = -\dfrac{\pi}{2}$ ou $\dfrac{3\pi}{2}$	1	
Donc $i_2(t) = \dfrac{\omega M I_1}{R_S+R_2}\,sin(\omega t)$	Bonus : 1	3 + 1 bonus
Q11 Puissance instantanée :		
$P(t) = R_s i_2^2 = R_S\dfrac{\omega^2 M^2 I_1^2}{(R_S+R_2)^2}sin^2(\omega t)$	2	
Puissance moyenne : $<P>= R_s i_2^2 = R_S\dfrac{\omega^2 M^2 I_1^2}{2(R_S+R_2)^2}$	1	
A.N. $<P>= 1.18.10^{-7}\,W$	2	5
Q12 La longueur d'onde associée à l'onde électromagnétique dans le vide est :		
$\lambda = \dfrac{c}{f} \approx 3.10^4 m$	1	

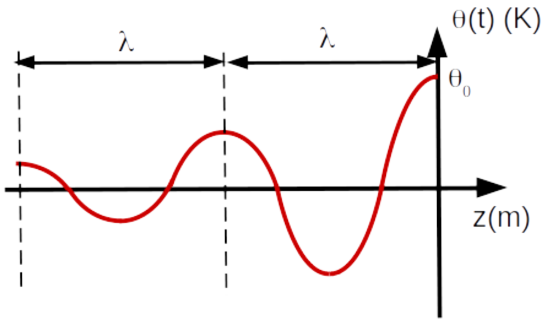
Elle est donc très supérieure aux distances mises en jeu dans le problème, ce qui autorise à négliger la propagation de cette onde. On est donc dans le cadre de l'ARQP	2 Bonus : 1	 3 + 1 bonus
Q13 Le coefficient d'induction mutuelle M est directement proportionnel à la perméabilité du milieu dans lequel se trouve la bobine réceptrice. $M = \frac{1}{2}\mu_0 \pi a^2 N_1 N_2 \frac{R^2}{(R^2+d^2)^{3/2}}$ Donc remplacer μ_0 par μ permet théoriquement de multiplier M par μ_r et donc la puissance aux bornes de R_S par μ_r^2 ! Inconvénient : L_2 n'est plus négligeable devant $R_S + R_2$ et l'impédance de la bobine va limiter ce gain.	 3 Bonus :1	 3+1 bonus
Q14 Pour augmenter la puissance disponible aux bornes de R_S il peut augmenter la surface de la bobine réceptrice ou le nombre de spires car cela va avoir des conséquences néfastes sur le poids et l'encombrement de la montre. On ne peut pas non plus les rapprocher beaucoup plus (2 mm) Il reste la valeur de I_1qu'on peut augmenter mais gare à l'effet Joule ! C'est donc la fréquence de $i_1(t)$ qui est le paramètre sur lequel on peut jouer le plus facilement, en l'augmentant (mais les impédances des bobines vont augmenter avec la fréquence...)	1 1 1 1	 4
Q15 (Bonus) 300 $mA.h$ sous 1,5 V : on doit fournir 300.10^{-3} x 1,5 x 3600 = 1620 Joules à la montre pour la recharger. Avec la valeur de la puissance trouvée précédemment, on trouve un temps de charge de $1620/1.18.10^{-7} = 1.3.10^{10}$ secondes, soit 436 ans environ... c'est un peu excessif Avec un matériaux ferromagnétique on divise théoriquement ce temps par 5000^2 et on tombe à 549 secondes, soit un peu plus de 9 mn (mais dans le même temps L_2 augmente beaucoup, ce qui relativise ce gain. Il parait donc indispensable de mettre la bobine réceptrice dans un milieu ferromagnétique	 1 1 1 1	 4
	Total	51+3 bonus

Partie 2 : Capteur de déplacement sans contact	Points	Total question
Q16 Equation de Maxwell - Gauss : $\vec{\nabla}.\epsilon\vec{E} = \rho = 0$ car il n'y a aucune charge volumique dans le système (Bonus +1 pour quiconque précise que dans les armatures métalliques, il n'y a que des charges surfaciques) Tout plan contenant l'axe (Oz) est un plan de symétrie de la distribution de charges. On constate une invariance en x et y, donc $\vec{E}$ ne dépend pas se x ni de y. Dans ces conditions $\vec{\nabla}.\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial E_z}{\partial z} = 0$ Donc $E_z = constante$.	1+1 (justif) 1 (sym+inv) 1	 4

Q17 : Relations de passage : $\vec{n}.(\vec{E}_{\epsilon 0} - \vec{0}) = \dfrac{\sigma_{\epsilon 0}}{\epsilon_0}\vec{u}_z$ (car le champ est nul dans les armatures métalliques), donc $\vec{E}_{\epsilon 0} = \dfrac{\sigma_{\epsilon 0}}{\epsilon_0} = \dfrac{2Q_{\epsilon 0}}{LW\epsilon_0}$ car la surface des armatures est $S_{\epsilon_0} = \dfrac{LW}{2}$ et $\sigma_{\epsilon 0} = \dfrac{Q_{\epsilon 0}}{S\epsilon_0}$. Le même raisonnement est valable côté ϵ : $\vec{E}_\epsilon = \dfrac{\sigma_\epsilon}{\epsilon_0} = \dfrac{2Q_\epsilon}{LW\epsilon}$	2	2
Q18 : En faisant circuler le champ électrique entre les deux électrodes, on a : $U = \int_0^e \vec{E}.\vec{dl} = E_{\epsilon 0}dz = \dfrac{Q_{\epsilon 0}}{LW\epsilon_0}e$, d'où on tire : $C_{\epsilon 0} = \dfrac{\epsilon_0 WL}{2e}$ De la même façon : $C_\epsilon = \dfrac{\epsilon WL}{2e}$ Les deux condensateurs sont branchés en parallèle, Donc $C_1 = C_{\epsilon 0} + C_\epsilon = \dfrac{LW}{2e}(\epsilon_0 + \epsilon)$	1 1 2	4
Q19 : Par analogie avec la partie précédente, et les surfaces mises en regard étant identiques, on peut écrire : $C_2 = C_1 = \dfrac{LW}{2e}(\epsilon_0 + \epsilon)$	1	1
Q20 : Si la pièce bouge d'une distance x, alors les surfaces des différentes parties des condensateurs change : $C_1' = \dfrac{\epsilon_0 W(\frac{L}{2} + x)}{e} + \dfrac{\epsilon W(\frac{L}{2} - x)}{e} = \dfrac{W}{2e}(L(\epsilon_0 + \epsilon) + 2x(\epsilon_0 - \epsilon) = C_1 + \dfrac{W}{e}x(\epsilon_0 - \epsilon)$	2	
$\Delta C_1 = \dfrac{W}{e}x(\epsilon_0 - \epsilon)$ De la même façon : $C_2' = \dfrac{\epsilon_0 W(L - x)}{2e} + \dfrac{\epsilon W(L + x)}{2e} = \dfrac{W}{2e}(L(\epsilon + \epsilon_0) + 2x(\epsilon - \epsilon_0) = C_2 + \dfrac{W}{e}x(\epsilon - \epsilon_0)$ $\Delta C_2 = \dfrac{W}{e}x(\epsilon - \epsilon_0)$ Ainsi : $\Delta C_1 = -\Delta C_2$	1 1 1 1	6
Q21 : bonus. C'est un double pont diviseur de tension $V_m = V_B - V_A$ avec $V_B - V_D = \dfrac{U}{2}$ et $V_A - V_D = \dfrac{Z_2}{Z_2 + Z_1}$, donc $V_m = \dfrac{U}{2} - \dfrac{Z_2}{Z_2 + Z_1}U$	2	Bonus : 2
Q22 : Pour une position quelconque de la pièce, sachant que : $Z_1 = \dfrac{1}{jC_1'\omega}$ et $Z_2 = \dfrac{1}{jC_2'\omega}$ $V_m = \dfrac{U}{2} - U\dfrac{\frac{1}{jC_2'\omega}}{\frac{1}{jC_2'\omega} + \frac{1}{jC_1'\omega}} = \dfrac{U}{2} - U\dfrac{\frac{1}{C_2'}}{\frac{1}{C_2'} + \frac{1}{C_1'}} = U(\dfrac{1}{2} - \dfrac{C_1'}{C_1' + C_2'}) = \dfrac{U}{2}(\dfrac{C_2' - C_1'}{C_2' + C_1'})$ $V_m = \dfrac{U}{2}\dfrac{C_2 + \Delta C_2 - C_1 - \Delta C_1}{C_2 + \Delta C_2 + C_1 + \Delta C_1} = \dfrac{U}{2}\dfrac{-2\Delta C_1}{2C_1} = -\dfrac{U}{2}\dfrac{\Delta C_1}{C_1}$	1 2 2	5
Q23 : Avec les données des exercices précédents : $V_m = \dfrac{U}{2}\dfrac{\frac{W}{e}x(\epsilon - \epsilon_0)}{\frac{LW}{2e}(\epsilon + \epsilon_0)} = \dfrac{x(\epsilon - \epsilon_0)}{L(\epsilon + \epsilon_0)}U$	2	2
Q24 : $C_2 = C_1 = 2.1\,pF$ Quand la pièce se déplace de 1cm $\Delta C_1 = -1.4\,pF$ (accepter uneQ variation positive) Avec une limite de tension de $1\,mV$, on peut détecter :	2 1	

$x_{min} = 10^{-3}\frac{L}{U}\frac{\epsilon + \epsilon_0}{\epsilon - \epsilon_0} = 3.10^{-6}\,m$, donc $x_{min} = 3\,\mu m$	2	
On gagnera peu à augmenter ϵ (à cause du rapport $\frac{\epsilon_0 + \epsilon}{\epsilon_0 - \epsilon}$), on ne peut donc que choisir un voltmètre plus tensible ou à la rigueur augmenter U (qui est déjà fort).	Bonus : 2	5 + 2 bonus
Total		29+4 bonus

Partie 3 : Champ électromagnétique dans une plaque métallique		
Q25 : Partant de l'équation de conservation de la chaleur et en remplaçant $\vec{j}$ par $\lambda\vec{\nabla}\theta$		
$\frac{\partial}{\partial t}(\rho C_p \Theta) + \vec{\nabla}.(\lambda(\vec{\nabla}\theta)) = 0 \Rightarrow \frac{\partial}{\partial t}(\rho C_p \Theta) + \Delta\theta = 0$		
Qu'on peut ré-écrire : $\frac{\lambda}{\rho C_p}\Delta\theta + \frac{\partial\Theta}{\partial t} = 0$, donc $D = \frac{\lambda}{\rho C_p}$	1	
L'analyse des dimensions donne D en $W.m^{-1}K^{-1}kg^{-1}m^3J^{-1}kg\,K$ soit en : $W.J^{-1}.m^2$ et donc en m^2s^{-1} (ne donner les points que pour l'expression finale faisant apparaître les secondes)	1	2
Q26 : On introduit cette onde dans l'équation d'onde de la question précédente, avec : $\Delta\underline{\theta} = -k^2\underline{\theta}$, et $\frac{\partial\theta}{\partial t} = j\omega\underline{\theta}$, il vient :		
$-k^2 D\underline{\theta} + j\omega\underline{\theta} = 0$	1	
Donc $-k^2 - j\omega 0 = 0 \Rightarrow k^2 = -\frac{j\omega}{D}$^	1	
Les valeurs possibles de k sont donc :		
$k = \pm\sqrt{\frac{\omega}{D}}e^{-j\frac{\pi}{4}} = \pm\sqrt{\frac{\omega}{D}}(cos(\frac{\pi}{4}) - j\,sin(\frac{\pi}{4})) = \pm\sqrt{\frac{\omega}{2D}}(1-j)$	1	
On a donc $A = \sqrt{\frac{\omega}{2D}}$	1	4
Q27 : En incorporant la valeur de k dans l'expression de l'onde harmonique on obtient :		
$\underline{\theta} = \theta_0 e^{j(\omega t + A(1-j)z)} = \theta_0 e^{+Az}e^{j(\omega t + Az)}$ ou $\underline{\theta} = \theta_0 e^{j(\omega t - A(1-j)z)} = \theta_0 e^{-Az}e^{j(\omega t - Az)}$	1	
La seconde expression ne convient pas car elle ne se propage pas dans le bon sens. De plus, si on se souvient que $z < 0$, alors son amplitude diverge. Il faut donc choisir la première expression.	1	
Expression réelle de l'onde :$\theta = \theta_0 e^{Az}cos\,(\omega t + Az)$	1	
Il s'agit d'une onde **harmonique (**ou **monochromatique**), **progressive**, se propageant le long des **z décroissants**, **uniforme amortie.**	2	
(compter 0,5 par terme pris dans cette liste, à concurrence de 2 points)		
Sa vitesse est $v = \frac{\omega}{A} = \sqrt{2D\omega}$	1	6
Q28 : D'après l'expression de l'onde : $\delta = \frac{1}{A} = \sqrt{\frac{2D}{\omega}}$	1	1
Q29 Si on choisit de représenter en fonction du temps, alors on doit fixer la position. Ici $z = 0$, l'expression de l'onde devient :		
$\underline{\theta} = \theta_0 e^{j(\omega t)}$, ce qui dans l'espace réel correspond à $\theta(t) = \theta_0 cos(\omega t)$	1	
Il suffira donc de représenter une fonction cosinus de phase initiale nulle et d'amplitude θ_0(enlever 1 point par information manquante sur le graphe : unités, phase à l'origine, amplitude, période).	2	
On fixe maintenant t= $2T$.		
L'expression de l'onde devient :		
$\underline{\theta}(t) = \theta_0 e^{+Az}e^{j(\omega 2T + Az)} = \underline{\theta}(t) = \theta_0 e^{+Az}e^{j(2\pi + Az)} = \theta_0 e^{Az}e^{jAz}$	2	
Ce qui donne dans l'espace réel : $\theta(t) = \theta_0 e^{Az}\,cos(Az)$		

 Point à enlever si : * Il n'y a pas deux périodes exactement (-1) * La température à l'origine est fausse (-1) * La courbe est représentée pour z>0 (-3, donc pas de points) * La courbe n'est pas amortie (-3, donc pas de points)	3	8
Q30 : Il faut pouvoir calculer la valeur numérique de la profondeur δ. On doit donc estimer les valeurs manquantes de manière plausible. **Evaluation de la période** T : une année = $3,15.10^7\,s$, ce qui correspond à $\omega = 2.10^{-7} rd.s^{-1}$, et donc à $\delta = 3.2\,m$ L'onde est atténuée à 95% au bout de 3δ, donc son amplitude est égale à 0.05θ_0 au bout de 9.6 m. Si on ne veut pas creuser trop profondément, il faut définir une **tolérance de température** et connaître **l'amplitude thermique** à laquelle la cave est soumise (c'est à dire la valeur de θ_0). Par exemple, si on ne veut pas que la température bouge de plus de 0.5° à Villeurbanne, où l'amplitude thermique peut atteindre $40\,K$, en enterrant la cave à 9.6 m, on a une variation de 0.05 x 40 = 2 degrés, c'est trop. A 4δ, on est à $0.01\,\theta_0 = 0.4$ K, c'est bon, mais la cave est alors à 12.8 m en dessous du sol... On peut comparer avec les fluctuations journalières en considérant $T = 8.64.10^4\,s$, mais $\delta_{journée} \ll \delta_{année}$ donc les calculs sur l'année peuvent servir de référence d'autant que les fluctuations sont plus grandes à l'année qu'au jour le jour. Donner tous les points pour une estimation plausible de δ et une discussion censée sur la tolérance de température incluant la connaissance de l'amplitude thermique.		4
Total partie 3 :		25
Total général :		105+ 7 bonus