End-of-Semester Physics Exam

The three exercises are independent. Hand-written formula sheet (one double-sided A4 page) and calculator in exam mode authorized. Un-justified answers may not be taken into account. Make sure your work is well presented and readable.

Formulae and useful constants :
Gradient in cylindrical coordinates : $\overrightarrow{\nabla} V = \overrightarrow{\mathrm{grad}}(V) = \frac{\partial V}{\partial r}\overrightarrow{u_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\overrightarrow{u_\theta} + \frac{\partial V}{\partial z}\overrightarrow{u_z}$
Divergence in cylindrical coordinates : $\overrightarrow{\nabla} \cdot \overrightarrow{A} = \mathrm{div}\overrightarrow{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$
Rotational in cylindrical coordinates :
$\overrightarrow{\nabla} \times \overrightarrow{A} = \overrightarrow{\mathrm{rot}}\ \overrightarrow{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\overrightarrow{u_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\overrightarrow{u_\theta} + \left(\frac{1}{r}\frac{\partial}{\partial r}(r\ A_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\overrightarrow{u_z}$
$\varepsilon_0 = \frac{1}{36\pi}.10^{-9}\,\mathrm{F\,m^{-1}}$; $\varepsilon = \varepsilon_r\varepsilon_0$; $\mu_0 = 4\pi \cdot 10^{-7}\mathrm{H\,m^{-1}}$
Conduction current in an ohmic conductor of conductivity γ : $\overrightarrow{j}_c = nq\overrightarrow{v} = \gamma\overrightarrow{E}$
Volume power density dissipated by Joule effect : $w_J = \gamma\overrightarrow{E}^2$

Exercise 1: parallel-plate capacitor - From the static case to the quasi-static assumption ($\simeq$ 10 points)

We consider a parallel-plate capacitor which metal plates are two disks, parallel, having the same axis Oz and of surface S. The medium located between the plates is an insulator (of conductivity $\gamma = 0$), of permittivity ε and of magnetic permeability μ_0.
With the capacitor initially discharged, it is placed in a circuit consisting of an ideal voltage generator of constant emf $U_0 > 0$, and of a series resistance R.
At the end of charging, the upper armature is charged with positive charge Q_0.
Numerical values : $S = 12\ \mathrm{mm}^2$, $e = 2\ \mathrm{mm}$, $U_0 = 10\ \mathrm{V}$

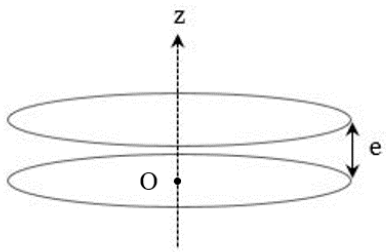


Figure 1: Parallel-plate capacitor with circular plates

Question 1-1: *Draw the electrical diagram used to charge this capacitor. Be sure to specify the direction of connection of the fem U_0 during charging.*

Part I : the capacitor is assumed to be fully charged

Question 1-2: *Specify, without calculation, the voltage $u_c = V(z = e) - V(z = 0)$ between the capacitor's plates after charging (no calculation needed here).*

Question 1-3: *Study of the topography of $\overrightarrow{E}$ between the plates (symmetry(ies), invariance(s) and consequence(s)).*

Question 1-4: *Same question if, in addition, disks can be considered as infinite planes in the study of symmetries and invariances.*

For the rest of the exercise we'll assume that the capacitor plates are very large compared to the interplate distance e, so that edge effects can be neglected (the symmetry and invariance properties are those of two disks of infinite radius). We'll use the usual direct orthonormal cylindrical coordinate system $(\overrightarrow{u_r}, \overrightarrow{u_\theta}, \overrightarrow{u_z})$.

Question 1-5: *Show that $\overrightarrow{E}$ between the plates is uniform .*

Question 1-6: *Derive the expression of $\overrightarrow{E}$ as a function of U_0.*

Question 1-7: *Determine the expression of $\overrightarrow{E}$ as a function of Q_0. Deduce the expression of the capacitance C of this parallel-plate capacitor.*

Question 1-8: *In the case where the same potential difference U_0 is imposed between the armatures, how does the charge carried by the capacitor plates vary when ε varies?*

Question 1-9:
Numerical application : determine the values of $\|\overrightarrow{E}\|$ and of Q_0 in the two cases $\varepsilon = \varepsilon_0$ and $\varepsilon = 4\varepsilon_0$.

Question 1-10: *[BONUS] Illustrate with a diagram the microscopic process at the origin of this difference in the two cases. $\varepsilon = \varepsilon_0$ and $\varepsilon = 4\varepsilon_0$.*

Part II : the capacitor is in the process of being charged (it is not completely charged yet)

Important note : throughout this section, it is never useful or necessary to determine the expression of $q(t)$.

At a time t when the charge is not complete:

Question 1-11: *Give and justify the expression of the Maxwell-Ampere equation valid between the capacitor plates. Specify the type(s) of current(s) involved between the plates during the charging phase.*

Question 1-12: *Give and justify the topography of the magnetic field that comes up between the capacitor's plates during charging. To do this, study the symmetries and invariances of currents in the inter-plate space with the cylindrical coordinate system.*

Question 1-13: *Prove that $\overrightarrow{B}(r = 0) = \overrightarrow{0}$.*

Question 1-14: *Prove that*

$$\overrightarrow{B} = kr\frac{dq(t)}{dt}\overrightarrow{u_\theta}$$

where k is a constant. The expression of k has to be determined as a function of variables among ε, μ_0 et S. Determine also its unit.

Question 1-15: *Calculate the total magnetic energy W_b stored in the capacitor at the time t.*

Question 1-16: *Deduce the self-inductance L of this capacitor.*

Question 1-17: *Numerical application: calculate L. Comment upon it.*

Part III : search for the $\left(\overrightarrow{\mathbf{E}}_{\mathbf{tot}}, \overrightarrow{\mathbf{B}}_{\mathbf{tot}}\right)$ fields present in the capacitor in forced sinusoidal regime

We are now interested in the case where the charge carried by the upper plate of the capacitor varies sinusoidally according to the following law

$$q(t) = Q_0 \cos\omega t, \quad \text{corresponding to the complex form} \quad \underline{q}(t) = Q_0\, \mathrm{e}^{j\omega t}$$

Question 1-18: *Deduce from the questions 7 and 14 the expressions of the complex representations, noted $\overrightarrow{\underline{E}}_0(t)$ and $\overrightarrow{\underline{B}}_1(t)$ respectively, of the electric and magnetic fields that exist between the plates as a function of Q_0, ω, S, ε and μ_0 and r.*

Question 1-19: *To which result does the Maxwell-Faraday equation lead in this case?*

We assume that $\overrightarrow{\underline{E_1}}(t)$ is the field to be added to $\overrightarrow{\underline{E_0}}(t)$ so that $\overrightarrow{\underline{E}}_{\text{tot}}(t) = \overrightarrow{\underline{E}}_0(t) + \overrightarrow{\underline{E}}_1(t)$ verifies Maxwell-Faraday equation. We look for a particular solution of Maxwell's equations which verifies $\overrightarrow{\underline{E}}_1(t) = \underline{E}_{1,z}(r,t)\, \overrightarrow{u}_z$ and $\overrightarrow{\underline{E}}_{\text{tot}}(r=0,t) = \overrightarrow{\underline{E}}_0(t)$.

Question 1-20: *Justify that the assumption made on the direction of $\overrightarrow{\underline{E}}_1$ is acceptable.*

Question 1-21: *Determine $\underline{E}_{1,z}(r,t)$.*

Question 1-22: *What is the name of the physical phenomenon behind the existence of $\overrightarrow{\underline{E}}_1$?*

Question 1-23: *Determine the expression for the complex representation of the total current density vector $\overrightarrow{\underline{j}}_{tot}$ that exists between the plates.*
Compare with the solution given in question 11. Any comments?

Question 1-24: *What equation must be satisfied by the correction $\overrightarrow{\underline{B}}_2$ to be added to $\overrightarrow{\underline{B}}_1$ if we want to take $\overrightarrow{\underline{E}}_1(r,t)$ into account?*

Question 1-25: *[BONUS] Do you think that $\left(\overrightarrow{\underline{E}}_0 + \overrightarrow{\underline{E}}_1, \overrightarrow{\underline{B}}_1 + \overrightarrow{\underline{B}}_2\right)$ are the complex representations of the total electromagnetic field $\left(\overrightarrow{\underline{E}}_{tot}, \overrightarrow{\underline{B}}_{tot}\right)$ that settles in between the plates? Explain your answer.*

Exercise 2: Induction-based position sensor (problem situation) ($\simeq$ 3.5 points)

Consider the direct orthonormal Cartesian base $(\overrightarrow{u_x}, \overrightarrow{u_y}, \overrightarrow{u_z})$ linked to the laboratory reference framework, with $\vec{u}_z$ the ascending vertical.

A position sensor can be realized using the following system:

A first coil $\mathscr{B}_R$ (not represented in Fig.2) is connected to a movable element (Rotor) which can rotate around the vertical axis Oz. We want to measure its angular position θ. This coil is supplied with a current i_R.

In the fixed part (Stator) surrounding the Rotor, we place two identical coils $\mathscr{B}_{S1}$ and $\mathscr{B}_{S2}$, with N turns of surface area s, and orthogonal axes Ox and Oy respectively. They are in an open circuit, so no current flows through them.

Moreover, the Ox axis of $\mathscr{B}_{S1}$ coincides with that of the Rotor when it is in its zero angular position.

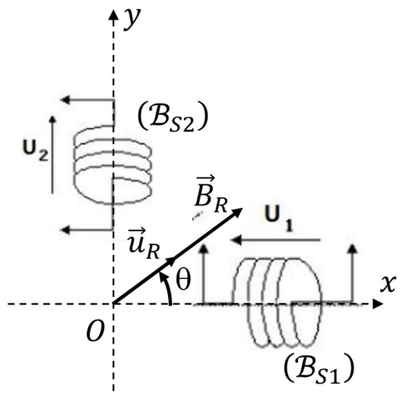


Figure 2: Principle of an induction-based position sensor

The surface area s of the coils $\mathscr{B}_{S1}$ and $\mathscr{B}_{S2}$ is small and the coils are close to the Rotor. We therefore consider that the magnetic field $\overrightarrow{B_R}$ sent by the coil $\mathscr{B}_R$ through the coils $\mathscr{B}_{S1}$ and $\mathscr{B}_{S2}$ is uniform and is in the direction of the Rotor axis. Let's take $\overrightarrow{B_R} = B_R(i_R)\overrightarrow{u_R} = Ki_R(t)\overrightarrow{u_R}$.

The voltages $u_1(t)$ and $u_2(t)$ respectively across each of the two stator coils, as defined in figure 2, are measured on both channels of an oscilloscope (input impedance considered infinite), when the current i_R is a sinusoidal current $i_R(t) = I_M \cos(\omega t)$.

Question 2-1: *The Rotor is at rest, but in any angular position θ to be determined. Show that the position θ of the Rotor can be deduced from the measurement of the voltages u_1 and u_2 and give the expression (or expressions, if there are more than one) for determining θ as a function of u_1 and u_2.*

Question 2-2: *What measurements should you take with your oscilloscope?*

Question 2-3: *What additional phenomenon occurs when the rotor rotates?*

Question 2-4: *Explain why this method could still give an approximation of the instantaneous position $\theta(t)$ of the Rotor, even if the Rotor rotates at constant angular velocity $\dot{\theta} = \Omega$, as long as its angular velocity verifies $\Omega \ll \omega$.*

Question 2-5: *[BONUS] Suggest an alternative protocol for finding the $\theta(t)$ position of the Rotor when the Rotor is in motion.*

Exercise 3: Magnetoresistance effet in Corbino disk ($\simeq$ 6.5 points)

Points in space are identified by their coordinates (r, θ, z) in the cylindrical coordinate system $(\vec{u}_r, \vec{u}_\theta, \vec{u}_z)$.
The Corbino disk is a **hollow** cylinder (for r<a) of axis (Oz), small height c and of inner and outer radii a et b respectively. It is made (for a<r<b) of a conductive material of electrical conductivity γ. The electron density n of the conductor is uniform.
This aim of this exercise is to study the effects of the application of a magnetic field on the electrical properties of the system.

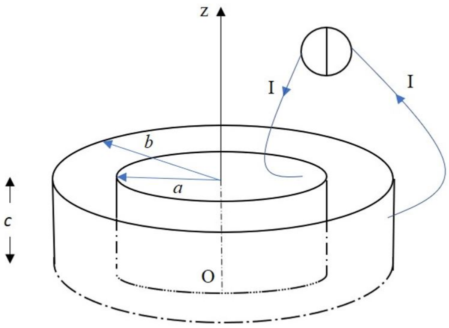


Figure 3: System called Corbino disk

I- Study of electrical conduction in the absence of a magnetic field

Remark : The index $_0$ corresponds to the calculated parameters in absence of magnetic field.
A current source supplies the disk with a current of intensity I. Due to the geometry of the Corbino disk, it is assumed that the current flows with a radial vector current density $\vec{j_0}$ (collinear to $\vec{u_r}$). We consider also that $\vec{j_0}$ is invariant respecting to θ and z.

At a point $M(a < r < b, \theta, 0 < z < c)$ inside of the conductor :

Question 3-1: *With the help of a sketch, calculate $\vec{j_0}(M)$ as a function of the current I, r and c.*

Question 3-2: *Deduce the electric field $\vec{E_0}(M)$ by recalling Ohm's local law.*

Question 3-3: *Determine the expression of the velocity $\vec{v_0}(M)$ of the electrons in movement in the conductor.*

Question 3-4: *Calculate the potential difference $U_0 = V(r = a) - V(r = b)$ between the inner and outer surfaces (or walls) of the Corbino disk.*

Question 3-5: *Deduce the resistance R_0 of the system in absence of magnetic field.*

II- Resistance change in the presence of a magnetic field

The Corbino disk is now placed in a uniform and constant magnetic field oriented along $\vec{u_z}$ $\vec{B}(M) = B\,\vec{u_z}$.
Even if the disk is not in movement, we assume that the study of the conduction inside of the disk in presence of the magnetic field $\vec{B}(M)$ can be carried-out using the principle of motional induction. We assume also that the motional induction is caused by only the radial component of the velocity of electrons $\vec{v}_e = v_r \vec{u}_r$.
In the presence of the magnetic field, once steady state is established, we assume that the radial component of the velocity of electrons in presence of $\vec{B}(M)$ remains unchanged compared to the case where $\vec{B}(M) = \vec{0}$ i.e. $\vec{v}_e = v_r \vec{u}_r = \vec{v_0}(M)$.

Question 3-6: *Calculate the induced electromotive field $\vec{E_m}(M)$.*

Question 3-7: *Deduce the induced e.m.f.* e_m.

Question 3-8: *Calculate the induced current density vector* $\overrightarrow{j_m}(M)$ *circulating inside of the disk, using Ohm's local law.*

Question 3-9: *Prove that the induced current* I_m *corresponding to the flux of* $\overrightarrow{j_m}(M)$ *through a surface to be defined, is given by :*

$$I_m = \frac{\mu I B}{2\pi} \ln\left(\frac{b}{a}\right)$$

where μ *is a constant characteristic of the conductor. The constant* μ *is called* ***electrical mobility*** *of the electrons in this material.* ***Not to be confused with magnetic permeability. You will express*** μ ***as a function of the electrical parameters of the conductor.***

Question 3-10: *We define the induced resistance* $R_m = \dfrac{e_m}{I_m}$. *Calculate the expression of* R_m. *Justify the homogeneity of the result by dimensional analysis.*

We quantify the influence of the application of the magnetic field by the magnetoresistance coefficient defined by :

$$MR = \frac{P_m}{P_0}$$

where P_0 and P_m correspond to the powers dissipated by Joule effect due to the two currents densities $\overrightarrow{j_0}(M)$ and $\overrightarrow{j_m}(M)$ respectively.

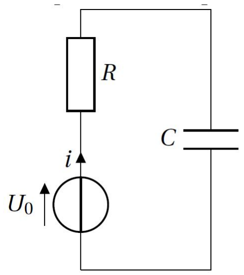
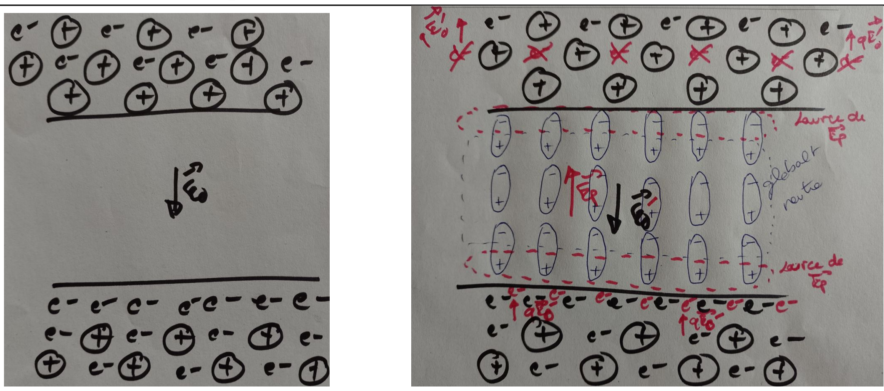
Question 3-11: *Determine* P_0 *using two different methods.*

Question 3-12: *Show that :*

$$MR = \mu^2 B^2$$

Question 3-13: *Numerical Application : Calculate* MR *for the semiconductor material InAs having electrons mobility* $\mu = 3.3\ \mathrm{m^2 V^{-1} s^{-1}}$ *at* $300\ \mathrm{K}$ *and in the presence of magnetic field of modulus* $B = 0.2\ \mathrm{T}$. *Comment.*

Note: this system is usually used to measure the mobility of charge carriers μ in semiconductor materials, in the presence of a known magnetic field.

Exercice 1 : Condensateur plan : de la statique à l'ARQS (39,5/80 + bonus 7,5)		/39,5
Q1. U_0 vers le haut (pointe vers $z=e$) (uniquement si on peut faire le lien à l'armature supérieur)  	0,5	
Q2. $U_c = U_0$	0,5	
Q3. invariance par rotation. $(M, \vec{u}_r, \vec{u}_z)$ de symétrie des charges, donc de $\vec{E}(M)$ et $E_\theta = 0$: $\vec{E}(M) = E_r(r,z)\vec{u}_r + E_z(r,z)\vec{u}_r$	0,5+1 0,5	
Q4. $\forall \underline{M}$: on peut maintenant utiliser les plans suivants de symétrie des charges, donc de $\vec{E}$: (xMz) donc $E_y(M) = 0$ et (yMz), donc $E_x(M) = 0$ avec de plus invariances par toutes translations // à Ox ou Oy : $\vec{E}(M) = E_z(z)\vec{u}_z$	0,5 1 0,5	
Q5. M-G, en tenant compte des invariances et en l'absence de charges entre les armatures : $div\vec{E} = 0 \Leftrightarrow \dfrac{\mathrm{d}E_z}{\mathrm{d}z} = 0$, et $E_z = cste$: E_z est bien uniforme. (ou encore l'équation de Poisson entre les armatures vérifie $\Delta V = 0 = \dfrac{\mathrm{d}^2V}{\mathrm{d}z^2}$, puisque pas de charge entre les armatures et $V(z)$ uniquement. Par intégration il vient donc $V(z) = az+b$, et $\vec{E} = \overrightarrow{grad}(-V) = \dfrac{\mathrm{d}(-V)}{\mathrm{d}z}\vec{u}_z \Rightarrow E_z(z) = -a = cste$)	1,5	
Q6. Par circulation le long de (Mz) : $U_0 = \displaystyle\int_0^e \mathrm{d}V = \int_0^e \overrightarrow{grad}(V)\cdot\vec{u}_z\mathrm{d}z = \int_0^e (-E_z)\mathrm{d}z \Leftrightarrow E_z = -\dfrac{U_0}{e}$	1,5 (−1 signe fx)	
Q7.Th. de Coulomb en $z=e$: $E_z = -\dfrac{\sigma}{\varepsilon} = -\dfrac{Q_0}{\varepsilon S}$, d'où avec Q6) : $E_z = -\dfrac{Q_0}{\varepsilon S} = \dfrac{U_0}{e} \Rightarrow C = \dfrac{Q_0}{U_0} = \dfrac{\varepsilon S}{e}$	1,5+1 (−1 signe fx)	
Q8. $Q_0 = CU_0 = \dfrac{\varepsilon S}{e}U_0$ est une fonction croissante (linéaire - non exigée) de ε.	0,5	
Q9. $\Vert\vec{E}\Vert = \vert E_z\vert = \frac{U_0}{e} = \frac{10\mathrm{V}}{2\times10^{-3}\mathrm{m}} = 5000\,\mathrm{Vm^{-1}}$, qui ne dépend que de U_0 et de e, et reste inchangé. $C_0 = \dfrac{\varepsilon_0 S}{e} = 0,053\,\mathrm{pF} = 5,3\times10^{-14}\,\mathrm{F}$ d'où $Q_0 = C_0U_0 = 5,3\times10^{-13}\,\mathrm{C}$ $C' = 4C_0 = 2,1\times10^{-13}\,\mathrm{F}$ pour $\varepsilon = 4\varepsilon_0$, et $Q' = 4Q_0 = 2,1\times10^{-12}\,\mathrm{C}$ (même faux cohérent)	1 1 0,5	
Q10. 	bonus (0,5fig. ε_0 +1,5 2è fig.)	
Q11. Le diélectrique est isolant : pas de courant de conduction. Par contre ici $E_z(t)$ et donc $\vec{j}_{tot} = \vec{j}_D = \dfrac{\partial \varepsilon E_z}{\partial t}\vec{u}_z \neq 0$. Le courant présent entre les armatures est le courant de déplacement et M-A s'écrit $\overrightarrow{rot}\left(\dfrac{\vec{B}}{\mu_0}\right) = +\dfrac{\partial\varepsilon E_z}{\partial t}\vec{u}_z$ (autres expressions : voir aussi Q14)	0,5 1	
Q12. $\vec{j}_{tot} = \varepsilon\dfrac{\mathrm{d}E_z}{\mathrm{d}t}\vec{u}_z$ est uniforme et vertical entre les armatures : $\forall M$ entre les armatures,	1	

$(M,\vec{u}_r,\vec{u}_z)$ est plan symétrie des courants, donc plan d'antisymétrie de $\vec{B}(M,t)$ et $\vec{B}(M,t)=B_\theta(r,z,t)\vec{u}_\theta$, puisque de plus il y a invariance par rotation d'axe Oz (accepter inv. sur r aussi)	1 1	
Q13. Par application à (xOz) et (yOz), on trouve $B_x(r=0,z)=0=B_y(r=0,z)$ respectivement. Comme on avait déjà $B_z(r,z)=0$, finalement $\vec{B}(r=0,z)=\vec{0}$.	1	
Q14. Avec $q(t)$ à la place de Q_0, il vient $E_z=-\dfrac{q(t)}{\varepsilon S}$ et $\vec{j}_{tot}=-\dfrac{\varepsilon}{\varepsilon S}\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\vec{u}_z$: M-A s'écrit $-\dfrac{\mu_0}{S}\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\vec{u}_z=-\dfrac{\partial B_\theta}{\partial z}\vec{u}_r+\dfrac{1}{r}\dfrac{\partial}{\partial r}(r\,B_\theta)\vec{u}_z$. On a donc $\dfrac{\partial B_\theta}{\partial z}=0$: $B_\theta(r,t)$ uniquement et $\dfrac{\partial(rB_\theta)}{\partial r}=-r\dfrac{\mu_0}{S}\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}$. D'où $rB_\theta=-\dfrac{\mu_0}{2S}r^2\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}+f(t)$. $f(t)=0$ par application à $r=0$ et $\vec{B}=-r\dfrac{\mu_0}{2S}\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\vec{u}_\theta$ (implique $\vec{B}(r=0)=\vec{0}$, mais pas utile dans la dém.) et $k=-\dfrac{\mu_0}{2S}$. Par la suite, compter juste tous les calculs cohérents qui en découlent (par ex. avec k)	1 0,5 (0 si $\overrightarrow{rot}$ incomplet) 2	
Q15. $W_b=\iiint\left(\dfrac{\vec{B}^2}{2\mu_0}\right)d\tau$ à intégrer sur le volume de l'espace inter-armature. $\dfrac{\vec{B}^2}{2\mu_0}d\tau=\dfrac{\mu_0}{8S^2}r^2\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2\mathrm{d}r r\mathrm{d}\theta\mathrm{d}z$ et $W_b=2\pi e\dfrac{\mu_0}{8S^2}\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2\displaystyle\int_0^{r_{max}}r^3\mathrm{d}r=\pi e\dfrac{\mu_0}{4S^2}\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2\dfrac{r_{max}^4}{4}$ $\left(=\pi e\dfrac{k^2}{\mu_0}\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2\dfrac{r_{max}^4}{4}\right.$ avec k). Avec $S=\pi r_{max}^2$, $W_b=\pi e\dfrac{k^2}{\mu_0}\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2\dfrac{S^2}{4\pi^2}=\pi e\dfrac{\mu_0}{4S^2}\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2\dfrac{S^2}{4\pi^2}=e\dfrac{\mu_0}{16\pi}\left(\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}\right)^2$	0,5 1,5 0,5	
Q16. $i=\dfrac{\mathrm{d}q(t)}{\mathrm{d}t}$ et $W_b=\dfrac{1}{2}Li^2$, d'où $L=-\dfrac{ek^2S^2}{2\pi\mu_0}=\dfrac{e\mu_0}{8\pi}$	1,5	
Q17. AN : $L=10^{-10}$ H, très faible	1+0,5	
Q18. $\underline{\vec{E}}_0=-\dfrac{Q_0}{\varepsilon S}\mathrm{e}^{j\omega t}\vec{u}_z$ et $\underline{\vec{B}}_1=jr\omega kQ_0\mathrm{e}^{j\omega t}\vec{u}_\theta=-jr\omega\dfrac{\mu_0 Q_0}{2S}\mathrm{e}^{j\omega t}\vec{u}_\theta$	0,5+1	
Q19. $\overrightarrow{rot}\left(\underline{\vec{E}}_0\right)=-\dfrac{\partial\underline{\vec{B}}_1}{\partial t}\Leftrightarrow\vec{0}=-j^2\omega^2r\dfrac{\mu_0Q_0}{2S}\mathrm{e}^{j\omega t}\vec{u}_\theta$. M-F n'est pas vérifiée : $\left(\underline{\vec{E}}_0,\underline{\vec{B}}_1\right)$ ne peut pas être le champ élmg entre les armatures	1 0,5	
Q20. Les liens entre les symétries de $\underline{\vec{E}}_{tot}$ et celles de sa source $-\dfrac{\partial\underline{\vec{B}}_1}{\partial t}$ sont du même type qu'entre $\vec{B}$ et les courants, puisque même type d'équation aux dérivées partielles : $\forall M$ entre les armatures, $(M,\vec{u}_r,\vec{u}_z)$ est un plan d'antisymétrie de $-\dfrac{\partial\underline{\vec{B}}_1}{\partial t}$, donc plan de symétrie de $\underline{\vec{E}}$, et $\underline{E}_\theta(M)=0$: c'est bien vérifié avec $\underline{\vec{E}}=\underline{E}_z\vec{u}_z$ (bonus : mais on ne montre pas que $\underline{E}_r=0$ aussi) (Le raisonnement suivant est aussi à valoriser, même si incomplet : d'après $\overrightarrow{rot}(\vec{E})=-\dfrac{\partial\vec{B}_1}{\partial t}$, sa source $-\dfrac{\partial\vec{B}_1}{\partial t}$ tourne autour de $\vec{E}$ et $\vec{E}=E_z\vec{u}_z$	bonus(1) 2+bonus(0,5) (1sur les 2)	
Q21. $\underline{\vec{E}}_{tot}=\underline{\vec{E}}_0(t)+\underline{E}_{1,z}(r,t)\vec{u}_z$ avec $\underline{\vec{E}}_0(t)$ uniforme impliquent $\overrightarrow{rot}(\vec{E})=-\dfrac{\partial E_{1,z}}{\partial r}\vec{u}_\theta$. D'où M-F $\Leftrightarrow\dfrac{\partial\underline{E}_{1,z}}{\partial r}=+r\omega^2\dfrac{\mu_0Q_0}{2S}\mathrm{e}^{j\omega t}$ et $\underline{E}_{1,z}=r^2\omega^2\dfrac{\mu_0Q_0}{4S}\mathrm{e}^{j\omega t}+E_a(t)$, avec $E_a=0$ car $\underline{\vec{E}}_{tot}(r=0,t)=\underline{\vec{E}}_0(t)$ (ou avec k : $\underline{E}_{1,z}=-r^2\omega^2\dfrac{kQ_0}{2}\mathrm{e}^{j\omega t}$).	1 1,5	
Q22. C'est l'induction statique.	0,5+0,5	
Q23. Toujours pas de courant de conduction, et $\underline{\vec{j}}_{tot}=\underline{\vec{j}}_D=\dfrac{\partial\varepsilon\underline{E}_{tot,z}}{\partial t}\vec{u}_z$ avec $\underline{E}_{tot,z}=\underline{E}_{0,z}+\underline{E}_{1,z}=\left(-\dfrac{Q_0}{\varepsilon S}+r^2\omega^2\dfrac{\mu_0Q_0}{4S}\right)\mathrm{e}^{j\omega t}$: $\underline{j}_{tot,z}=j\omega\left(-\dfrac{Q_0}{S}+r^2\omega^2\dfrac{\varepsilon\mu_0Q_0}{4S}\right)\mathrm{e}^{j\omega t}=\underline{j}_0+\underline{j}_1$. $\vec{E}$ modifié, d'où $\underline{\vec{j}}_{tot}=\underline{\vec{j}}_D$ modifié, ou bien on vérifie la loi de Lenz qualitative	1 0,5	

Une correction en ω^2 s'ajoute au 1er terme. On peut remarquer que $\left\|\frac{\underline{j}_1}{\underline{j}_0}\right\| = \frac{1}{4}r^2\omega^2\varepsilon\mu_0 = \frac{\varepsilon_r}{4}\frac{r^2\omega^2}{c^2}$. La correction ne devient significative qu'à HF : à BF tq $\omega \ll \omega_{max} = \frac{c}{r_{max}}$ on a $\left\|\frac{\underline{j}_1}{\underline{j}_0}\right\| \ll 1$	bonus(2)	
Q24. $\overrightarrow{rot}\left(\frac{\underline{\vec{B}}_1+\underline{\vec{B}}_2}{\mu_0}\right) = +\frac{\partial\varepsilon\left(\underline{E}_{z,0}+\underline{E}_{z,1}\right)}{\partial t}\vec{u}_z = \left(\underline{j}_0+\underline{j}_1\right)\vec{u}_z$. Comme $\overrightarrow{rot}\left(\frac{\underline{\vec{B}}_1}{\mu_0}\right) = +\frac{\partial\varepsilon\underline{E}_{z,0}}{\partial t}\vec{u}_z = \underline{j}_0\vec{u}_z$, il reste $\overrightarrow{rot}\left(\frac{\underline{\vec{B}}_2}{\mu_0}\right) = +\frac{\partial\varepsilon\underline{E}_{z,1}}{\partial t}\vec{u}_z = \underline{j}_1\vec{u}_z = j\omega^3 r^2\frac{\varepsilon\mu_0 Q_0}{4S}\,\mathrm{e}^{j\omega t}\,\vec{u}_z$ (avec k : $\overrightarrow{rot}\left(\frac{\underline{\vec{B}}_2}{\mu_0}\right) = -j\omega^3 r^2\frac{\varepsilon k Q_0}{2}\,\mathrm{e}^{j\omega t}\,\vec{u}_z$)	0,5 1	
Q25. $\underline{\vec{B}}_1(t)$ est à la source de la correction $\underline{\vec{E}}_1(t)$ à $\underline{\vec{E}}_0(t)$; le terme correctif $\underline{\vec{E}}_1(t)$ est à la source de l'ajout d'un terme correctif $\underline{\vec{B}}_2(t)$, qui de même devrait être à la source d'un nouveau terme correctif $\underline{\vec{E}}_2(t)$, qui va conduire à un terme correctif suivant $\underline{\vec{B}}_3(t)$, et ainsi de suite. Pour trouver $\left(\underline{\vec{E}}_{tot}(t), \underline{\vec{B}}_{tot}(t)\right)$, on doit déterminer la suite des termes correctifs $\left(\underline{\vec{E}}_n(t), \underline{\vec{B}}_n(t)\right)$ et en déduire la série $\left(\sum_0^\infty \underline{\vec{E}}_n(t), \sum_1^\infty \underline{\vec{B}}_n(t)\right) = \left(\underline{\vec{E}}_{tot}(t), \underline{\vec{B}}_{tot}(t)\right)$.	bonus(1) bonus(1)	

Exercice 2 : capteur de position par induction (14/80 + bonus 2)		/14
Q1. Les bobines statoriques sont le siège d'une induction statique : $u_1 = -\frac{\mathrm{d}\Phi_1}{\mathrm{d}t}$ et $u_2 = -\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}$. Comme $\vec{B}_R$ est considéré comme uniforme, on a simplement $\Phi_1 = N\varphi_1 = NB_R\vec{u}_R \cdot s\vec{n}_1$. Vu le sens de l'enroulement de la bobine $\mathscr{B}_{S1}$ sur la figure 2, on doit prendre $\vec{n}_1 = +\vec{u}_x$ pour respecter la définition de u_1 sur la figure 2. uniquement si justifié : On a donc $\Phi_1 = NsKi_R(t)\cos\theta$ et $u_1 = NsKi_M\omega\cos\theta\sin(\omega t)$ De même $\Phi_2 = NB_R\vec{u}_R \cdot s\vec{n}_2$ avec $\vec{n}_2 = +\vec{u}_y$ pour respecter le sens de l'enroulement de $\mathscr{B}_{S2}$ et la définition de u_2 sur la figure 2. uniquement si justifié : Soit $\Phi_2 = NsKi_R(t)\sin\theta$ et $u_2 = NsKi_M\omega\cos\theta\sin(\omega t)$ Finalement on a par exemple $\frac{u_2}{u_1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$	1 1 1 1,5 0,5 1 1 1	
Q2. Il suffit donc de mesurer $u_1(t)$ et $u_2(t)$. De plus, d'après l'étude précédente, on a trouvé $u_1(t)$ et $u_2(t)$ en phase et la meilleure précision pour $\tan\theta$ sera obtenue en déterminant la tension crête-crête des voies 1 et 2.	0,5 1	
Q3. Toujours induction statique mais 2 sources de variations de $\vec{B}$: $i_R(t)$ et $\theta(t)$	0,5	
Q4. On a toujours $\Phi_1 = NsKi_M\cos(\omega t)\cos\theta(t)$, d'où $u_1 = -NsKi_M\left(-\omega\sin(\omega t)\cos\theta(t) - \dot\theta\cos(\omega t)\sin\theta(t)\right)$. De même $u_2 = -NsKi_M\left(-\omega\sin(\omega t)\sin\theta(t) + \dot\theta\cos(\omega t)\cos\theta(t)\right)$ Le 2ème terme est d'ordre de grandeur négligeable si $\dot\theta = \Omega \ll \omega$: avec cette approximation, on retrouve la même loi que en Q1).	0,5 1 1 1 0,5	
Q5. [BONUS] on pourrait imaginer une alimentation du Rotor en courant continu : alors la seule variation dans le temps est lié à $\theta(t)$. On aurait $\Phi_1 = NsKi_M\cos\theta(t)$, $u_1 = NsKi_M\dot\theta\sin\theta(t)$, et de même $\Phi_2 = NsKi_M\sin\theta(t)$, $u_2 = -NsKi_M\dot\theta\cos\theta(t)$, et finalement $\tan\theta = -\frac{u_1}{u_2}$	bonus(2)	

Exercice 3 : disque de Corbino (26,5/80)	/26,5

Q1. $I_0 = \iint_{(S)} \overrightarrow{j_0}(M).\vec{n}\,dS$, avec $dS = rd\theta\ dz$, $\vec{n} = +\vec{u}_r$ dans le sens de la flèche de I, et $\overrightarrow{j_0}(M) = j_0(r)\overrightarrow{u_r}$ (radial, ne dépend ni de z ni de θ). (dont 0,5 pour explication sens de $+\vec{u}_r$) $I_0 = \int_0^c \int_0^{2\pi} j_0(r).rd\theta\ dz = 2\pi rc\, j_0(r) \Leftrightarrow j_0(r) = \dfrac{I_0}{2\pi rc}$ (dans la suite prendre en compte les calculs cohérents avec $\vec{j_0}$ trouvé)	1,5 1	
Q2. Loi d'Ohm Local : $\overrightarrow{j_0}(M) = \gamma\overrightarrow{E_0}(M)$ et $\overrightarrow{E_0}(M) = \dfrac{\overrightarrow{j_0}(M)}{\gamma} = \dfrac{I_0}{2\pi\gamma rc}\overrightarrow{u_r}$	1	
Q3. $\overrightarrow{j_0}(M) = n.q.\overrightarrow{v_0}(M) = -en\overrightarrow{v_0}(M)$, d'où $\overrightarrow{v_0}(M) = -\dfrac{\overrightarrow{j_0}(M)}{ne} = -\dfrac{I_0}{2\pi nerc}\overrightarrow{u_r}$	1	
Q4. $dV = -\overrightarrow{E_0}(M)\cdot\overrightarrow{d\ell}$ avec $\overrightarrow{d\ell} = dr\overrightarrow{u_r} + rd\theta\overrightarrow{u_\theta} + dz\overrightarrow{u_z}$ d'où $dV = -E_0(r).dr = -\dfrac{I_0}{2\pi\gamma c}\dfrac{dr}{r}$ $U_0 = V(r=a) - V(r=b) = \int_{V(r=b)}^{V(r=a)} dV = -\dfrac{I_0}{2\pi\gamma c}\int_b^a \dfrac{dr}{r} = \dfrac{I_0}{2\pi\gamma c}\int_a^b \dfrac{dr}{r} = \dfrac{I_0}{2\pi\gamma c}\ln\dfrac{b}{a}$ $(-1$ signe faux)	1,5 1,5	
Q5. On est en convention récepteur (vu les sens de calculs utilisés précédemment), d'où $U_0 = R_0 I_0 = \dfrac{I_0}{2\pi\gamma c}\ln\dfrac{b}{a} \Leftrightarrow R_0 = \dfrac{\ln\frac{b}{a}}{2\pi\gamma c}$ $(0,5$ si cohérent mais $R_0 < 0)$	0,5 1	
Q6. $\vec{E}_m = \vec{v}_e \wedge \vec{B} = -Bv_0(r)\vec{u}_\theta = \dfrac{IB}{2\pi nerc}\vec{u}_\theta$	1,5	
Q7. On utilise la circulation sur un cercle de rayon r e_{ind} (de sens $+\vec{u}_\theta$; non exigé) vérifie $e_{ind} = \int_{\theta=0}^{2\pi} \vec{E}_m\cdot\overrightarrow{d\ell}$ avec $\overrightarrow{d\ell} = rd\theta\vec{u}_\theta$. $e_{ind} = \int_{\theta=0}^{2\pi} \dfrac{IB}{2\pi nerc} r\,d\theta = \dfrac{IB}{nec}$	0,5 1 1	
Q8. Loi d'Ohm Local appliquée à $\vec{E}_m$: $\overrightarrow{j_m}(M) = \gamma\overrightarrow{E_m}(M) = \gamma\dfrac{IB}{2\pi nerc}\vec{u}_\theta$	1	
Q9. On calcule le flux de $\overrightarrow{j_m}$ à travers une section droite du matériau perpendiculaire à $\overrightarrow{j_m}$, donc à travers l'intersection du matériau par un plan méridien ($\theta = cste$) : (0,5 si surface utilisée pas explicite) Avec $\vec{n} = \vec{u}_\theta$, on calcule I_m (de flèche dans le sens de $\vec{u}_\theta$; non exigé), et $dS = dr\,dz$: $I_m = \int_{r=a}^b \int_{z=0}^c \dfrac{\gamma IB}{2\pi nerc}\vec{u}_\theta\cdot\vec{u}_\theta dr\,dz : I_m = \dfrac{\gamma IB}{2\pi ne}[\ln r]_a^b = \dfrac{\gamma IB}{2\pi ne}\ln\left(\dfrac{b}{a}\right)$, et $\mu = \dfrac{\gamma}{ne}$	1 1,5	
Q10. $R_m = \dfrac{e_{ind}}{I_m} = \dfrac{\frac{IB}{nec}}{\frac{\gamma IB}{2\pi ne}\ln\left(\frac{b}{a}\right)} = \dfrac{2\pi}{\gamma c\ln\left(\frac{b}{a}\right)}$ $(= \dfrac{2\pi}{\mu nec\ln\left(\frac{b}{a}\right)}$ si on garde $\mu)$ $(0,5$ si cohérent mais $R_m < 0)$ de dimension $dim(R_m) = \dfrac{1}{Ldim(\gamma)}$, cohérent avec $dim(R) = dim\left(\dfrac{\rho\ell}{S}\right) = \dfrac{L}{L^2 dim(\gamma)}$.	1 1,5	
Q11. $P_0 = R_0 I^2 = \dfrac{\ln\frac{b}{a}}{2\pi\gamma c}I^2$ 2ème méthode : en utilisant la densité volumique de puissance dissipée par effet Joule : $dP_0 = \gamma\vec{E}_0^2 d\tau$ avec $d\tau = dr\,rd\theta\,dz$. Il vient $dP_0 = \gamma\left(\dfrac{I}{2\pi\gamma c}\right)^2\dfrac{dr}{r}d\theta\,dz$ et $P_0 = \int_{r=a}^b\int_{\theta=0}^{2\pi}\int_{z=0}^c dP_0 = \dfrac{I^2}{2\pi\gamma c}\ln\left(\dfrac{b}{a}\right)$	1 1 1,5	
Q12. De même il vient (par ex.) $P_m = R_m I_m^2 = \dfrac{2\pi}{\gamma c\ln\left(\frac{b}{a}\right)}\left(\dfrac{\gamma IB}{2\pi ne}\ln\left(\dfrac{b}{a}\right)\right)^2 = \dfrac{\gamma I^2B^2}{2\pi n^2e^2c}\ln\left(\dfrac{b}{a}\right)$ d'où $MR = \dfrac{\frac{\gamma I^2B^2}{2\pi n^2e^2c}\ln\left(\frac{b}{a}\right)}{\frac{I^2}{2\pi\gamma c}\ln\left(\frac{b}{a}\right)} = \dfrac{\gamma^2B^2}{n^2e^2}$. On retrouve bien $MR = \mu^2B^2$ avec $\mu = \dfrac{\gamma}{ne}$	1 1,5	
Q13. AN : $MR = (3.3\times0.2)^2 = 44\%$, ce qui est significatif : l'effet Joule dissipé par l'effet de magnéto-résistance représente environ la moitié de celui en l'absence de B.	1+0,5	