| **Exercise 1 :** Direct Current Circuit (6/20 + bonus 0.5) | /6 |
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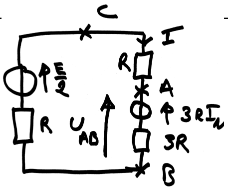
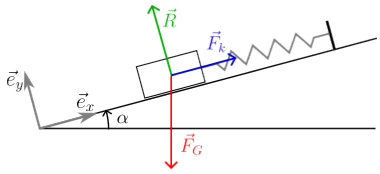
| Q1. After 4 transformations (switching to Norton for (E, 2R), combining 2 resistors in parallel, switching back to Thevenin on the left, switching to Thevenin for the real current source : 4 transformations with 0.5 pt each), we arrive at the following single-loop circuit :   | (Same total points for any other correct method) 2 |
| Applying Kirchhoff's Voltage Law (KVL) to this loop, we get : $E/2 - RI - RI - 3RI - 3RI_N = 0$, | 0.5 |
| which gives $I = \dfrac{E/2 - 3RI_N}{5R}$ | 0.5 |
| and $U_{AB} = 3RI_N + 3RI = 3RI_N + \dfrac{3}{5}(E/2 - 3RI_N) = \dfrac{3}{5}(E/2 + 2RI_N)$ | 1 |
| We verify that these two expressions are homogeneous (Bonus 0.5 total for the two verifications). | Bonus 0.5 |
| Q2. A dipole $\mathscr{D}$ operates as a receiver if the electrical power received by the dipole is positive, i.e., $U_{\mathscr{D}} I_{\mathscr{D}} > 0$ in Passive Sign convention (or alternatively, a dipole operates as a receiver if the power supplied by the dipole to the current is negative, i.e., $U_{\mathscr{D}} I_{\mathscr{D}} < 0$ in Active Sign convention). | 1.0 |
| Here, for the dipole "real current source," we look for the conditions that satisfy $U_{AB} I > 0$ (we are in load convention), i.e., $\dfrac{3}{5}(E/2 + 2RI_N)\dfrac{E/2 - 3RI_N}{5R} > 0$. With $E, I_N > 0$, it suffices that $E/2 - 3RI_N > 0 \Leftrightarrow E > 6RI_N$. (Non-mandatory remark : since the only active component in $\mathscr{D}$ is the ideal current source (ICS), one could have limited the analysis to the Operating Zone of the ICS.) | 1 |

| **Exercise 2 :** Capacitor Discharge (7/20 + bonus 2) | /7 |
|---|---|
| Analyze :<br>The capacitor (initially charged to a voltage $U_0$) discharges through a person modeled as a resistor $R$. Figure 2 shows the effects of the current as a function of contact time. The task is to determine the current evolution over time in this transient regime. | 0.5 |
| Modeling the experiment : Complete the discharge circuit diagram with all relevant variables ($U(t)$, $C$, $R$, and $i(t)$). | 0.5 |
| Realize :<br>Initial condition (IC) : continuity of the voltage across the capacitor terminals (following charging; a demonstration is optional and can be done by modeling $C$ as an open switch in steady state) : $U(t=0^+\text{s}) = U(t=0^-\text{s}) = U_0$. | 0.5 (+0.5 bonus for demonstration) |
| Study of discharge : Treat $C$ using the generator convention, leading to $i(t) \overset{(1)}{=} -C\dfrac{\mathrm{d}U}{\mathrm{d}t}$ ; treat $R$ using the load convention, giving $U \overset{(2)}{=} Ri(t)$. | 0.75 + 0.25 |
| Substituting (2) into (1) : $i(t) + RC\dfrac{\mathrm{d}i}{\mathrm{d}t} = 0$ (differential equation, DE). (Accept points if the DE is expressed for $u(t)$ : $u(t) + RC\dfrac{\mathrm{d}u}{\mathrm{d}t} = 0$.) | 0.5 |
| (DE) is a first-order homogeneous differential equation. A solution of the form $i = i_0\,\mathrm{e}^{rt}$ is tested in the DE, yielding $r + \dfrac{1}{RC} = 0$. | 0.5 |
| Additionally, at $t = 0^+$s, $U(0^+\text{s}) = U_0 = Ri(t=0^+\text{s})$, so $i(t=0^+\text{s}) = \frac{U_0}{R} = i_0\,\mathrm{e}^0$. Thus, $i = \frac{U_0}{R}\,\mathrm{e}^{-\frac{t}{\tau}}$ with $\tau = RC$. (Accept points for $u(t) = U_0\,\mathrm{e}^{-\frac{t}{\tau}}$ and $i(t) = -C\dfrac{\mathrm{d}U}{\mathrm{d}t} = \frac{U_0}{R}\,\mathrm{e}^{-\frac{t}{\tau}}$.) | 0.5 |
| Bonus : Verify homogeneity and long-term behavior (tends to zero as $t \to \infty$).<br>For a sign error resulting in an exponentially increasing solution at long times, grant full points for REA if the student identifies and corrects the error (3 points). If not, give 1/3 of the points if everything else is correct. | Bonus 0.5 |
| Validate :<br>The current is not constant, so to answer the question (nature of the perceived electrification), one must estimate a "time of electrification" and an "average current." Accept various interpretations, even slightly approximate ones, as long as they make sense. For example :<br>AN : $i_0 = \frac{100\text{V}}{1\text{k}\Omega} = 0.1\text{A} = 100\text{mA}$ and $\tau = 1\text{k}\Omega \times 500\mu\text{F} = 0.5\text{s}$. | 0.5 |
| Since $i(t)$ is decreasing, it is certain that we are not in the fibrillation zone. However, we are likely in the tetanization zone. | 1 |
| Bonus : Additionally, calculate $i(t=\tau) = 100\text{mA}\,\mathrm{e}^{-1} \simeq 37\text{mA}$. This shows that for at least 0.5s, the current exceeds 37mA, confirming we are in the tetanization zone. | Bonus 0.5 |
| Communicate : Clear reasoning and logical flow are evident in the student's work. | 0.5 |
| Discussion of the statement :<br>To compare to the given statement, begin by evaluating the electrical energy received : all energy stored in the capacitor is discharged through $R$, which therefore receives the energy $W_{elec} = \dfrac{1}{2}CU_0^2 \overset{AN}{=} 2.5\text{J}$. This is less than the 5J stated in the text. | 1 |
| One could argue that in the previous question, the electrification experienced was significant since it caused tetanization. The claim that an electrification below 5J has no effect on a person seems incorrect unless fibrillation is specifically required. | Bonus 0.5 |

| **Exercise 3 :** Mass-spring system on an inclined plane | /8 |
|---|---|
| Q1. In the Terrestrial reference frame, considered as Galilean, we study the system {block} :<br>The external forces are :<br>• The weight $\vec{F}_G = m\vec{g} = -mg\sin\alpha\,\vec{e}_x - mg\cos\alpha\,\vec{e}_y$<br>• The force due to the spring : $\vec{F}_k = +k(\ell-\ell_0)\vec{e}_x$<br>• The reaction force $\vec{R} = \vec{R}_N\vec{e}_y$ with $R_N > 0$ (no friction).<br><br>At equilibrium, we have $\vec{F}_G + \vec{F}_k + \vec{R} = \vec{0}$<br>By projection on $\vec{e}_x$, we get<br>$$-mg\sin\alpha + k(\ell-\ell_0) = 0 \Leftrightarrow k = \frac{mg\sin\alpha}{\ell-\ell_0}$$<br>Numerical application :<br>min/max method or differential method : $k = (1.24 \pm 0.10)$ N.m$^{-1}$ or $k = (1.2 \pm 0.1)$ N.m$^{-1}$<br>*For the differential method note that* $\frac{\Delta(\sin\alpha)}{\sin\alpha} = \frac{\Delta\alpha}{\tan\alpha}$ | 0.5<br>1<br>0.5<br>(scheme)<br>0.5<br>0.5<br>0.5 |
| Q2. If the block is moved to the right, the extension $\ell-\ell_0$ decreases, the spring force thus decreases. The other forces do not change, so that the resulting force along $\vec{e}_x$ is oriented towards the left (negative). The system goes back to its equilibrium position.<br>If the block is moved to the left, the spring force increases and the resulting force is positive along $\vec{e}_x$, bringing the block back to its equilibrium position.<br>The equilibrium position is therefore **stable** | 1 |
| Q3. Noting $\vec{R}_T$ and $\vec{R}_N$ the tangential and normal components of the reaction force, we have<br>$$\lVert\vec{R}_T\rVert \leqslant f\lVert\vec{R}_N\rVert$$ | 0.5 |
| Q4. The system is subjected to the same external force, but now the reaction force is $\vec{R} = \vec{R}_T + \vec{R}_N$, where $\vec{R}_N = R_N\vec{e}_y$ and $\vec{R}_T = R_T\vec{e}_x$.<br>At equilibrium, we have now<br>$$\begin{cases} -mg\sin\alpha + k(\ell-\ell_0) + R_T = 0 \\ -mg\cos\alpha + R_N = 0 \end{cases}$$<br>This gives $R_N = mg\cos\alpha$ and $k = (mg\sin\alpha - R_T)/(\ell-\ell_0)$<br>The extreme values $k$ can take are obtained for the extreme values of $R_T$, so that<br>$$k_{\min} = \frac{mg}{\ell-\ell_0}(\sin\alpha - f\cos\alpha), \quad k_{\max} = \frac{mg}{\ell-\ell_0}(\sin\alpha + f\cos\alpha)$$<br>Numerical application : $k_{\min} = 0.90$ N.m$^{-1}$ and $k_{\min} = 1.58$ N.m$^{-1}$<br>The range is quite large! | 0.25<br>0.5<br>0.25<br>0.5<br>0.25<br>0.25 |
| Q5. Friction is negligible if the term $f\cos\alpha$ is small compared to $\sin\alpha$.<br>This is the case if $f \ll \tan\alpha = 0.364$. Friction can thus be neglected in the case of steel on ice, but not in the other cases (small but not very small for steel/Teflon) | 0.5<br>0.5 |